Chapter 5 Pulse modulation

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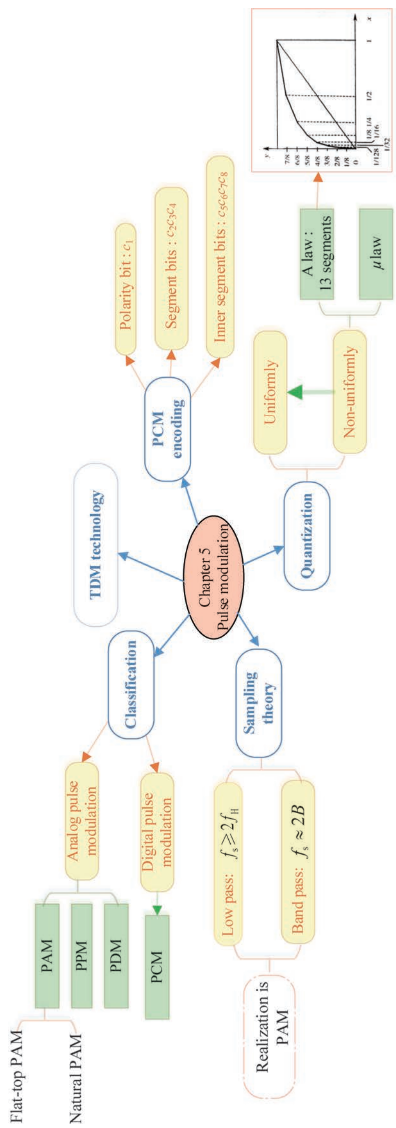


Mind map for Chapter 5

In Chapter 4, some parameters of a sinusoidal carrier wave are varied continuously in accordance with the message signal. In this chapter, some parameters of a pulse train are varied in accordance with the message signal. The carrier wave is a pulse, which is different from CW modulation.

The following issues will be discussed:

- Sampling, which is basic to all forms of pulse modulation.
- Pulse amplitude modulation (PAM,脉冲幅度调制) (In discrete amplitude form).
- Quantization (量化), after the process of sampling, in discrete form in both amplitude and time.
- Pulse-code modulation (PCM,脉冲编码调制), which is the standard method for the transmission of an analog message signal by digital means.
- TDM (时分复用).
- Delta modulation (DM/ΔM,Δ 调制/增量调制).

  Differential pulse-code modulation (DPCM,差分脉冲编码调制).

视频讲解

5.1 Sampling process

The process of digitizing an analog signal is illustrated in Figure 5.1.1. There are three main steps in the process of digitizing an analog signal: sampling, quantization and encoding.

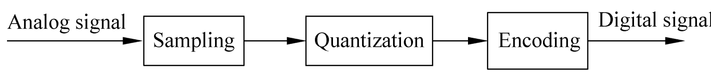


Figure 5.1.1 The process of digitizing an analog signal

1. The sampling theorem (the sampling of low-pass analog signals)

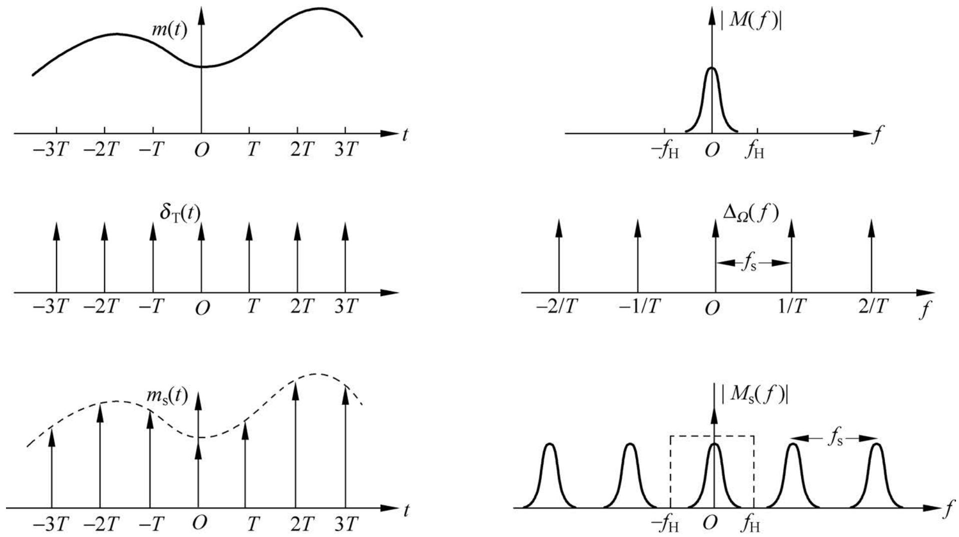
If the highest frequency of a continuous analog signal $s(t)$ is less than f_H, and if it is sampled by periodic impulses with interval time $T \leqslant 1/2f_H$, then $s(t)$ can be completely decided by these samples, as shown in Figure 5.1.2.

The ideal form of sampling process is called instantaneous sampling (瞬时采样). T_s is the sampling period, and $f_s = \dfrac{1}{T_s}$ is the sampling rate.

The derived process is in the following Table 5.1.1: let the spectra of $m(t)$, $\delta_T(t)$ and $m_s(t)$ be expressed by $M(f)$, $\Delta_\Omega(f)$ and $M_s(f)$. If the sampling frequency is lower than $2f_H$, the adjacent spectra will be superposed (重叠), hence the original signal spectrum $M(f)$ could not be separated correctly by LPF at the receiver.

Table 5.1.1 The derivation of sampling process

$m(t)$	$M(f)$
$\delta_T(t) = \delta(t - nT_s)$	$\Delta_\Omega(f) = \dfrac{1}{T}\sum_{n=-\infty}^{\infty}\delta(f - nf_s)$
$m_s(t) = m(t) \cdot \delta_T(t)$	$M_s(f) = M(f) * \Delta_\Omega(f) = \dfrac{1}{T}\sum_{-\infty}^{\infty}M(f - nf_s)$



(a) Time domain (b) Frequency domain

Figure 5. 1. 2 Sampling process

From Figure 5. 1. 2, the condition of restoration of original signal is $f_s \geqslant 2f_H$. The lowest sampling frequency $2f_H$ is called Nyquist sampling rate(奈奎斯特采样速率). The corresponding largest sampling time interval is called Nyquist sampling interval(奈奎斯特采样间隔).

2. The sampling of band-pass analog signals

The frequency band of band-pass signals is limited between f_L and f_H. $B = f_H - f_L$ is the bandwidth of the analog signal. Here, we only give the result. The sampling frequency f_s can be written as:

$$f_s = 2B + \frac{2KB}{n} = 2B\left(1 + \frac{K}{n}\right) \tag{5.1.1}$$

where n is the largest integer (整数) less than f_H/B, $0 < k < 1$. The relation between f_s and f_L is shown in Figure 5. 1. 3.

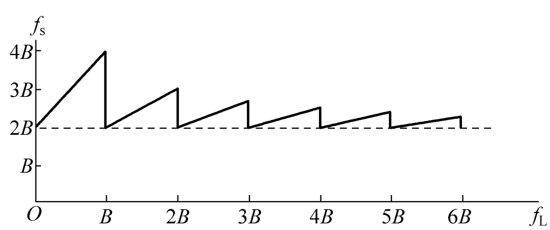


Figure 5. 1. 3 The relation between f_s and f_L

When $f_L = 0$, $f_H = 2B$, this is the sampling condition for the low pass filter.

When $f_L \gg 0$, $f_s \to 2B$, the signal is a narrow-band signal.

5.2 Analog pulse modulation（模拟脉冲调制）

There are three parameters of pulse: amplitude, width (duration) and position. Therefore, we can get three different pulse modulations: PAM (pulse amplitude modulation), PDM (pulse duration modulation) and PPM (pulse position modulation), as shown in Figure 5.2.1.

Although these modulations are discrete in time, they are still analog modulations. In order to convert an analog signal to a digital signal, quantizer should be used in the next step.

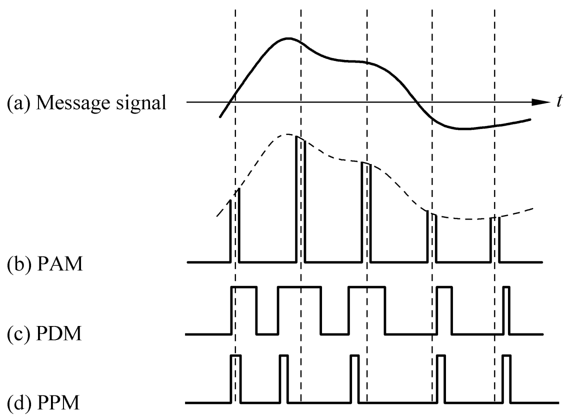


Figure 5.2.1 Three types pulse modulation

In this part, the PAM is introduced in detail. There are two types PAM: natural sampling (自然采样) and flat-top sampling (平顶采样).

1. Natural sampling PAM

As can be seen from Figure 5.2.2, the top of each pulse is natural.

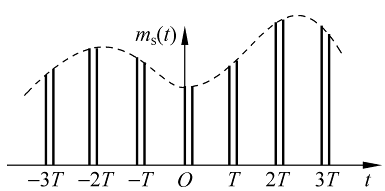


Figure 5.2.2 Natural sampling PAM

The differences between sampling and PAM is the period signal $s(t)$, which is the period pulses.

According to

$$m_s(t) = m(t) \cdot s(t)$$

Its corresponding Fourier transform is

$$M_s(f) = M(f) * S(f)$$

$$=\frac{A\tau}{T}\sum_{n=-\infty}^{\infty}\mathrm{sinc}(\pi n\tau f_{\mathrm{H}})M(f-2f_{\mathrm{H}})$$

Because the carrier wave is periodic rectangular pulses, the envelope of $M_s(f)$ is the sinc function. The original signal $M(f)$ can be recovered/reconstructed when $M_s(f)$ passes through a LPF at the receiver (see Figure 5.2.3).

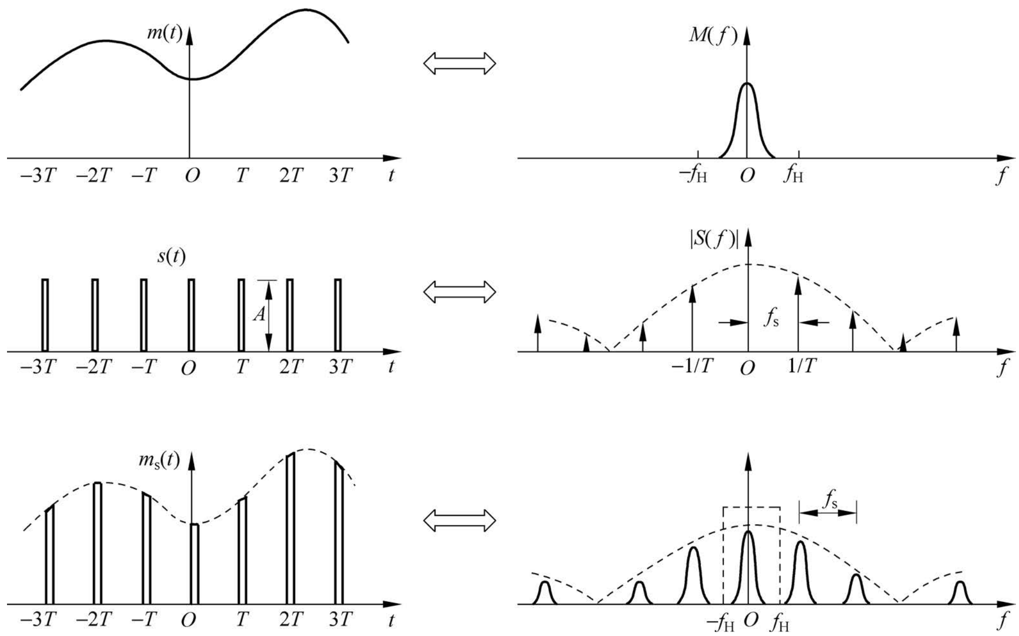


Figure 5.2.3 The waveform and spectrum of PAM

2. Flat-top sampling PAM

The top of PAM is flat, as shown in Figure 5.2.4.

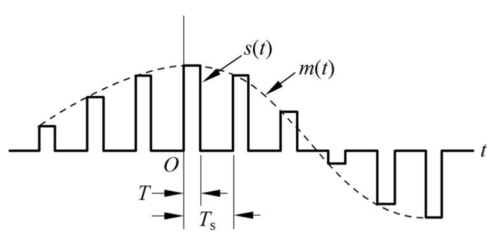


Figure 5.2.4 Flat-top sampling PAM

There are two operations involved in the generation of the PAM signal, as shown in Figure 5.2.5.

(1) Instantaneous sampling (瞬时采样).

The sampling rate f_s is choosed in accordance with the sampling theorem.

(2) Lengthening (保持) the duration of each sample so obtained to some constant value T.

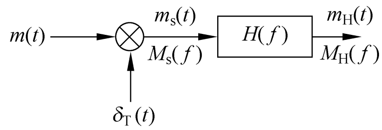


Figure 5.2.5 The principle of PAM

Where

$$h(t)=\begin{cases}1, & 0<t<T\\ \dfrac{1}{2}, & t=0, t=T\\ 0, & \text{otherwise}\end{cases}$$

$$H(f)=T\,\mathrm{sinc}(fT)\exp(-\mathrm{j}\pi fT)$$

In Table 5.2.1, the comparisons between time and frequency domain equations are given.

Table 5.2.1 The derivation of PAM

Time domain	Frequency domain
$m_s(t)=m(t)\cdot\delta_T(t)=\sum_{n=-\infty}^{\infty} m(nT_s)\cdot\delta(t-nT_s)$	$M_s(f)=M(f)*\Delta_n(f)=f_s\sum_{n=-\infty}^{\infty} M(f-nf_s)$
$m_H(t)=m_s(t)*h(t)$	$M_H(f)=M_s(f)\cdot H(f)=f_s\sum_{n=-\infty}^{\infty} M(f-nf_s)\cdot H(f)$

The problem is how to recover the original message signal $m(t)$. The first step is that $m_H(t)$ passes through a LPF(low pass filter). The spectrum of LPF output is equal to $M(f)H(f)$. Obviously, there is frequency distortion because of $H(f)$. This distortion may be corrected by connecting an equalizer in cascade(串联) with the LPF, as shown in Figure 5.2.6.

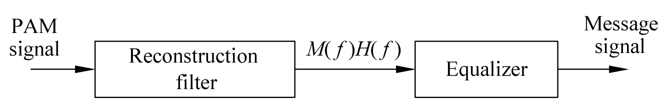


Figure 5.2.6 The reconstruction of message signal

The magnitude response of the equalizer is given by

$$\frac{1}{|H(f)|}=\frac{1}{T\,\mathrm{sinc}(fT)}=\frac{\pi f}{\sin(\pi fT)}$$

For a duty cycle(占空比) $T/T_s\leqslant 0.1$, the amplitude distortion is less than 0.5%, in which case the need for equalization may be omitted altogether.

视频讲解

5.3 Quantization process(**量化过程**)of sampled signal

Amplitude quantization: the process of transforming the sample amplitude $m(kT)$ of a message signal $m(t)$ at time $t=kT$ into a discrete amplitude $m_q(kT)$ taken from a finite set of possible amplitudes(有限个可能的取值). An example of quantization process is shown in Figure 5.3.1.

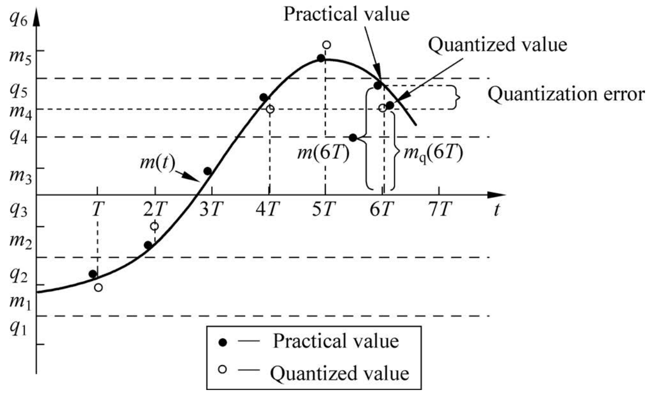


Figure 5.3.1 The quantization process

$m(kT)$ expresses the sampled signal input to a quantizer, $m_q(kT)$ expresses the quantized value of the output signal of this quantizer, as shown in Figure 5.3.2.

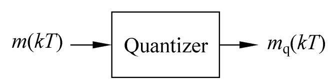


Figure 5.3.2 Quantizer

$q_1 \sim q_6$ are 6 possible levels of the quantized signal.

$m_1 \sim m_5$ are the boundary points of the quantization intervals.

$$m_q(kT) = q_i, \quad m_{i-1} \leqslant m(kT) \leqslant m_i \tag{5.3.1}$$

In this example, M intervals are with equal space, so it is called uniform quantization (均匀量化). If the M intervals are with unequal spaces, it is called nonuniform quantization (非均匀量化), which is based on the uniform quantization.

nonuniform quantization⇨uniform quantization

5.3.1 Uniform quantization (均匀量化)

Assume the magnitudes of the sampled signal is within $a \sim b$, and the number of quantization levels is M, so the quantization interval (量化间隔) is $\Delta\nu = \dfrac{b-a}{M}$ (also called step-size: 步长), and the boundary points of the quantization intervals are $m_i = a + i\Delta\nu$, $i = 0, 1, \cdots, M$.

Obviously, quantized output m_q is different from signal sample before quantization m_k, there is an error of quantizer. This error is usually called quantization noise (量化噪声) and signal power to quantization noise power ratio (信噪比) is used for estimating the influence of the error to the signal.

The average value N_q of the quantized noise power for uniform quantization can be expressed by

$$N_q = [(m_k - m_q)^2] = \int_a^b (m_k - m_q)^2 f(m_k) dm_k$$

$$= \sum_{i=1}^{M} \int_{m_{i-1}}^{m_i} (m_k - q_i)^2 f(m_k) dm_k \quad (5.3.2)$$

where $f(m_k)$ is the probability density of signal sample m_k

$$m_i = a + i\Delta\nu$$

$$q_i = a + i\Delta\nu - \frac{\Delta\nu}{2} \quad (5.3.3)$$

The average power of signal m_k can be expressed as

$$S = E(m_k^2) = \int_a^b m_k^2 f(m_k) dm_k \quad (5.3.4)$$

Example 5.3.1: Assume the number of quantization levels for a uniform quantizer is M, and the sample of the input signal has uniform probability density in the interval $[-a, a]$. Find the average signal to quantization noise ratio for the quantizer.

Solution: From equation(5.3.2), we can get

$$N_q = \sum_{i=1}^{M} \int_{m_{i-1}}^{m_i} (m_k - q_i)^2 f(m_k) dm_k = \sum_{i=1}^{M} \int_{m_{i-1}}^{m_i} (m_k - q_i)^2 \left(\frac{1}{2a}\right) dm_k$$

$$= \sum_{i=1}^{M} \int_{-a+(i-1)\Delta\nu}^{-a+i\Delta\nu} \left(m_k + a - i\Delta\nu + \frac{\Delta\nu}{2}\right)^2 \cdot \left(\frac{1}{2a}\right) dm_k$$

$$= \sum_{i=1}^{M} \left(\frac{1}{2a}\right) \cdot \left(\frac{\Delta\nu^3}{12}\right) = \frac{M(\Delta\nu)^3}{24a}$$

According to $M \cdot \Delta\nu = 2a$, so the quantization noise power is

$$N_q = \frac{(\Delta\nu)^2}{12}$$

The average power of signal is

$$S_0 = \int_{-a}^{a} m_k^2 \left(\frac{1}{2a}\right) dm_k = \frac{M^2}{12} \cdot (\Delta\nu)^2$$

The the average signal to quantization noise ratio is:

$$\frac{S_0}{N_q} = M^2 \quad \text{or} \quad \left.\frac{S_0}{N_q}\right|_{\text{dB}} = 20\lg M(\text{dB})$$

$\frac{S_0}{N_q}$ is increasing along with the increasing of number M. That is to say when the signal is small, signal to quantization noise ratio is also small. For the speech signal, there are lots of small components. To overcome this disadvantage, the nonuniform quantization is often used in practical applications.

5.3.2 Nonuniform quantization

A nonuniform quantizer is equivalent to passing the sampled signal through a compressor and then applying the compressed signal to a uniform quantizer, as illustrated in Figure 5.3.3.

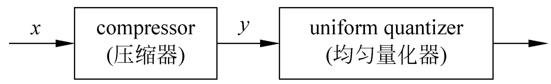


Figure 5.3.3 The equivalent process of nonuniform quantizer

Compression uses a nonlinear circuit to convert input voltage x to output voltage y:

$$y = f(x)$$

where x has nonuniform scale and y has uniform scale.

Theoretically, $f(x)$ should be a logarithm(对数)function. $f(x)$ needs to be properly modified in different requirements of the practical condition. There are two ITU recommended logarithm compression laws (对数压缩定律): A-law and μ-law, for telephone signal. The logarithm laws are give in Table 5.3.1.

Table 5.3.1 Logarithm compression laws

A-law	μ-law
$y=\begin{cases}\dfrac{Ax}{1+\ln A}, & 0<x\leqslant\dfrac{1}{A}\\ \dfrac{1+\ln Ax}{1+\ln A}, & \dfrac{1}{A}\leqslant x\leqslant 1\end{cases}$	$y=\dfrac{\ln(1+\mu x)}{\ln(1+\mu)}$
13 segments	15 segments
E1: China & European countries	**T1**: North American, Japan and Korea

A-law 13 segments method: A-law compression is a continuous smooth curve. It is difficult to be accurately achieved by electronic circuit, while the 13 segments method can be easily achieved by digital circuit approximately, which is plotted in Figure 5.3.4.

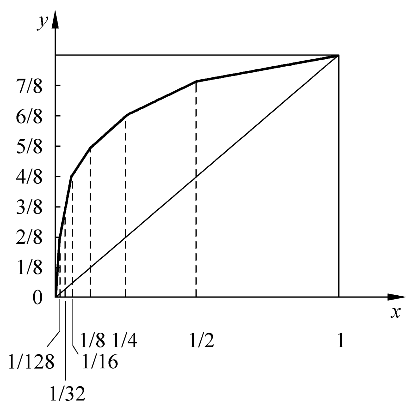


Figure 5.3.4 A-law 13 segments

In Table 5.3.2, the slope of each segment is given. As can be seen, the first and second segments have the same slope.

Table 5.3.2 Slope of each segment

No.(折线段号)	1	2	3	4	5	6	7	8
slope(斜率)	16	16	8	4	2	1	1/2	1/4

Why we called it 13 segments?

The input voltage of speech signal has positive and negative polarities. So the above compression curve is only one half of the practical characteristic curve of the compressor. There is another half in the 3rd quadrant（象限），so the curve is odd symmetry to the origin（关于原点奇对称）.

The slopes of the 1st and 2nd segments in 1st quadrant and 3rd quadrant are the same. These 4 segments compose a straight line. Therefore，it is called 13 segments.

Signal after quantization is a digital signal with discrete values. The next step is how to encode these discrete values.

视频讲解

5.4 PCM（pulse code modulation，脉冲编码调制）

PCM is the most basic form of digital pulse modulation.

PCM：a message signal is represented by a sequence of coded pulses which is accomplished by representing the signal in discrete form in both time and amplitude（PCM编码后的信号在时间和幅度上的取值都是离散的）.

The most often used code is to use the binary symbols representing discrete values，such as 0 and 1. The basic operations performed in the transmitter of a PCM system are sampling，quantizing and encoding，as illustrated in Figure 5.4.1. The quantizing and encoding operations are usually performed in the same circuit，which is called an analog-to-digital converter（A/D 转换器）. The basic operations in the receiver is the regeneration of the message signals：decoding and reconstruction.

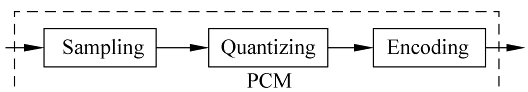


Figure 5.4.1 The PCM process

5.4.1 The principle of PCM

There are 8 bits in PCM code word（码字），which can satisfy the speech communication quality.

c_1—the polarity bit（极性位）$\begin{cases} +, c_1=1 \\ -, c_1=0 \end{cases}$；

$c_2c_3c_4$—segment bits（段落码）：000～111；

$c_5c_6c_7c_8$—inner-segment bits（段内码）：000～1111（16 inner-segments in each segment）.

Inner-segment bits are uniformly encoded according to quantization intervals，but quantization intervals of different segments are different.

Among these 8 segments，the lengths of 1st and 2nd segments are the shortest 1/128， and slopes of them are the largest, after it is equally divided into 16 sub-segments, the each sub-segment equals:

$$\Delta = \frac{1}{128} \times \frac{1}{16} = \frac{1}{2048}$$

which is called the smallest quantization interval(最小量化间隔).

If uniform quantization is used to keep the same dynamic range 1/2048, then 11 bits code word is necessary.

While for the PCM systems, only 7 bits (不含极性位) are used, which embody the advantage of nonuniform quantization.

The typical sampling rate of telephone signal is 8kHz ($\geqslant 2 \times 3.4$kHz). Thus the typical transmission bit rate of digital telephone is 64kb/s in PCM system. This rate has been adopted by ITU-T recommendation.

Example 5.4.1: Using A-law 13 segments encoding, if the sampling value is +1270, find the output of PCM encoder and the quantization error.

Answer: (1) First, since $+1270>0$, the polarity bit $c_1=1$;

(2) Second, 1270 is in the 8th segment, so $c_2c_3c_4=111$;

(3) Finally, determine which inner-segment 1270 is in. There are usually two methods. One is the successive comparison (逐次比较). Another one is direct calculation. The 8th segment is plotted in Figure 5.4.2.

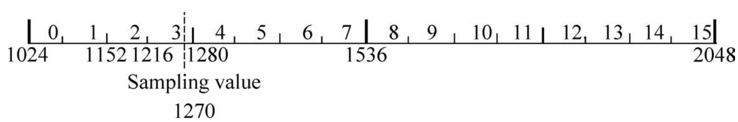


Figure 5.4.2 The quantization interval of 8th segment

Successive comparisons:

$$1270 < 1536, \quad c_5 = 0$$
$$1270 < 1280, \quad c_6 = 0$$
$$1270 > 1152, \quad c_7 = 1$$
$$1270 > 1216, \quad c_8 = 1$$

Direct calculation:

inner-segment quantization interval is $(2048-1024)/16=64$;

Since $(1270-1024)/64 \approx 3.8$, the inner segment coding is $c_5c_6c_7c_8=0011$.

Quantization error is: $1270-(1216+1280)/2=22\Delta$.

Therefore, the output of PCM encoder is 11110011 and the quantization error is 22Δ.

5.4.2 Noise in PCM system

The derivation signal to quantization noise ratio of the uniform quantizer has been discussed in Section 5.3.1.

$$S/N_q = M^2$$

When N bits binary code word is used for encoding, the above equation can be written as

$$S/N_q = 2^{2N} \quad (M = 2^N)$$

This equation shows that S/N_q of PCM system is only related to N and increases with N exponentially. For a low-pass signal, the sampling rate should be no less than $2f_H$, so in the PCM system, this is equivalent to not less than $2Nf_H$ b/s. The system bandwidth is at least $B = Nf_H$.

$$S/N_q = 2^{2(B/f_H)}$$

The S/N_q of PCM system increases with the bandwidth B exponentially.

5.4.3 Delta modulation（增量调制）

1. Principle

Delta modulation (DM or ΔM) provides a staircase approximation to the over-sampled (i.e., at a rate much higher than the Nyquist rate) version of the message signal, as illustrated in Figure 5.4.3.

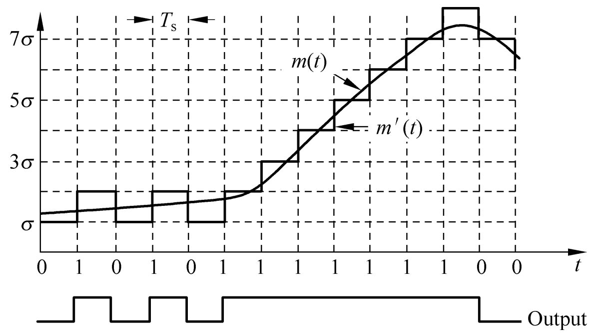


Figure 5.4.3 The waveform of DM

The difference between the input and the approximation is quantized into only two levels, namely, $\pm\Delta$, corresponding to positive and negative differences.

If the approximation falls below the signal at any sampling epoch, it is increased by Δ. If the approximation lies above the signal, it is diminished by Δ.

The principal (主要的) virtue of delta modulation is its simplicity. It may be generated by applying the sampled version of the incoming message signal to a modulator that involves a comparator (比较器), quantizer and accumulator interconnected as shown in Figure 5.4.4. The block labeled z^{-1} inside the accumulator represents a unit delay, that is, a delay equal to one sampling period.

The basic principle of Δ modulation is in the following:

$$e[n] = m[n] - m_q[n-1]$$

$$e_q = \Delta \operatorname{sgn}[e(n)]$$

$$m_q[n] = m_q[n-1] + e_q[n]$$

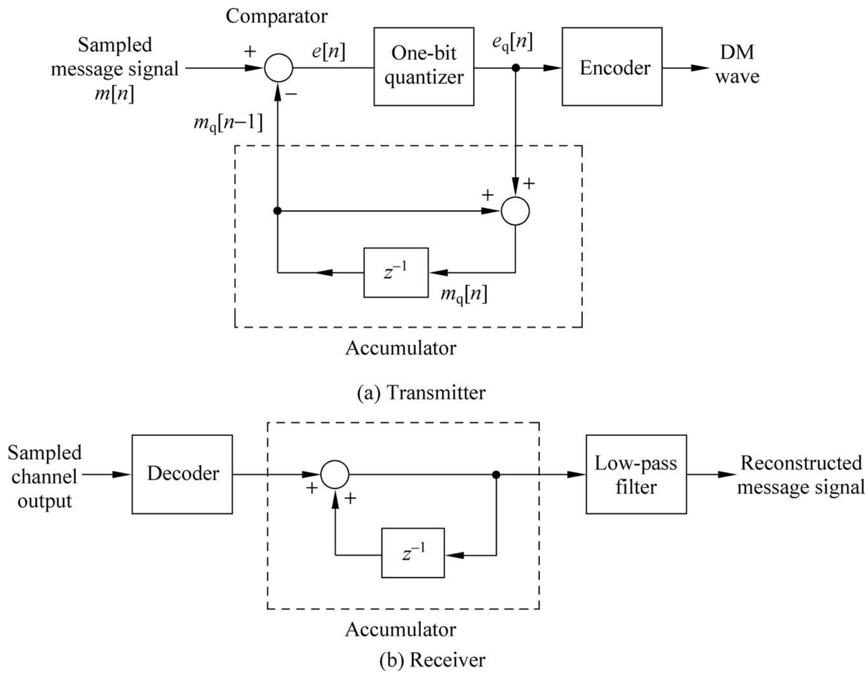


Figure 5.4.4 DM system

The quantizer output $m_q[n]$ is coded to produce the DM signal. The rate of information transmission is simply equal to the sampling rate $f_s=\frac{1}{T_s}$.

2. Quantization error

There are two types of quantization error: slope overload distortion and granular noise (see Figure 5.4.5).

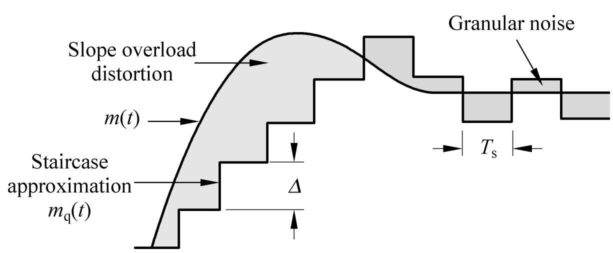


Figure 5.4.5 Two different forms of quantization error

If considered the maximum slope of the original input waveform $m(t)$, it is clear that in order for the sequence of samples $\{m_q[n]\}$ to increase as fast as the input sequence of samples $\{m[n]\}$ in a region of maximum slope of $m(t)$, we require that the following condition:

$$\frac{\Delta}{T_s} \geqslant \max\left|\frac{\mathrm{d}m(t)}{\mathrm{d}t}\right|$$

be satisfied. Otherwise, we find the step-size Δ is too small for the staircase approximation $m_q(t)$. In contrast to slope-overloaded distortion, granular noise is analogous (类似) to quantization noise in a PCM system.

5.4.4 DPCM（differential pulse code modulation，差分脉冲调制）

When a voice or video signal exhibits a high degree of correlation between adjacent samples（邻近的采样）the resulting PCM encoded signal will contain redundant information（冗余信息）.

In order to remove this redundancy before encoding, we use a more efficient coded signal, which is the basic idea of DPCM, as shown in Figure 5.4.6.

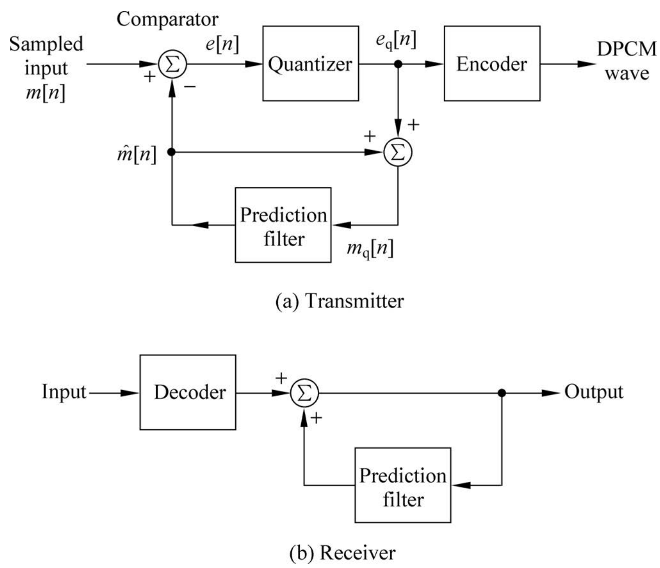


Figure 5.4.6 DPCM system

The input signal of the DPCM quantizer is

$$e[n]=m[n]-\hat{m}[n]$$

where $m[n]$ is the unquantized sample signal and $\hat{m}[n]$ is the prediction of $m[n]$.

The relationship between the output and input of the predictor is

$$\hat{m}=\sum_{i=1}^{q}a_i m_q\text{（线性预测）}$$

where p is the prediction order and a_i is the prediction coefficient（系数）. The predicted value is the weighted sum of previous p samples of the signal with quantization error.

If $p=1, a_1=1$ then $\hat{m}=m_{q-1}$. The predictor is simply a delay circuit, and the delay time is sampling interval T. If the quantizer is one-bit (two-level), the DPCM is ΔM, i.e. ΔM is the special case of DPCM.

5.5 TDM（time-division multiplexing，时分复用）

The concept of TDM is shown in Figure 5.5.1. Each input message signal is first restricted in a low-pass anti-aliasing filter. The functions of each part are described below.

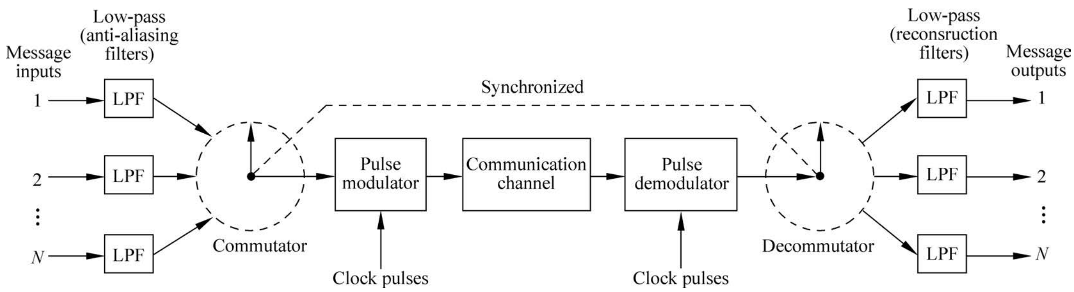


Figure 5. 5. 1 Block diagram of TDM system

LPF: restricts each input message in bandwidth and removes the nonessential frequencies.

Commutator (换向器): which is usually implemented using electronic switching circuitry. The function is to sequentially interleave these N samples inside. The sampling interval is T. It is the essence of TDM operation.

Pulse modulator: transform the multiplexed signal into a form suitable for transmission over the common channel.

Pulse demodulator: the reverse operation of pulse modulator.

Decommutator: operates in synchronism (同步) with commutator in the transmitter.

The multiplexing of digital signals is accomplished by using a bit-by-bit interleaving procedure with a selector switch that sequentially takes a bit from each incoming line and then applies it to the high-speed common line. At the receiving end of the system the output of this common line is separated out into its low-speed individual components and then delivered to their respective destination, as plotted in Figure 5. 5. 2.

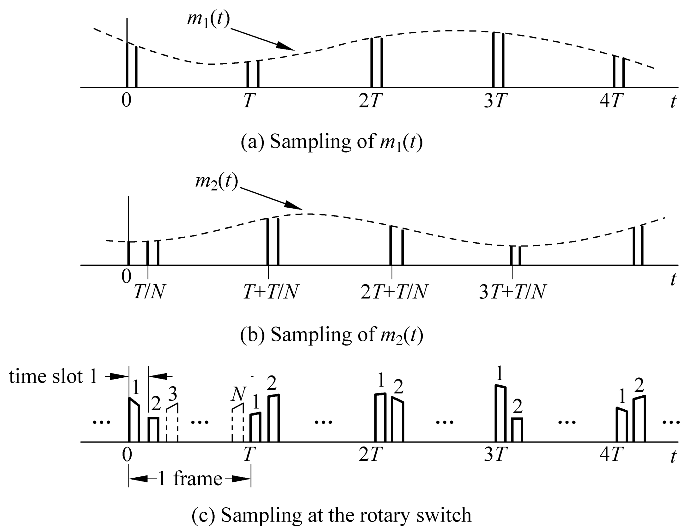


Figure 5. 5. 2 Principle of TDM

Summary and discussion

In this chapter，we introduced the process of analog signal digitization：sampling，quantization and encoding.

（1）Sampling，which operates in the time domain；the sampling process is the link between an analog waveform and its discrete-time representation，which also is an analog signal in amplitude.

（2）Quantization，which operates in the amplitude domain；the quantization process is the link between an analog waveform and its discrete-amplitude representation.

（3）Encoding，which operates in both time and amplitude domain，encoding process is the link between discrete-amplitude and binary representation.

The sampling process is based on the sampling theorem，which states that when an analog signal with frequency band within $(0, f_H)$ is sampled，the lowest sampling rate should not be less than Nyquist sampling rate $2f_H$. The sampled signal may be converted to different analog pulse modulation signals，including PAM，PDM and PPM. In TDM of several channels，signal processing usually begins with PAM. In order to make full use of the time interval of each sampling point，TDM is designed and applied in speech communication system.

There are two methods for the quantization of a sampled signal，one is uniform quantization，another one is nonuniform quantization. Nonuniform quantization is usually applied in speech signal with the logarithm characteristic recommended by ITU-T，i. e.，A-law and μ-law，which may effectively improve signal to quantization noise ratio，especially the small amplitude signal. European countries and China adopt A-law；North American countries and Japan as well as other counties and areas adopt μ-law. 13 segment and 15 segment methods are applied in digital circuits to achieve the A-law and μ-law quantization.

Signal after quantization is already a digital signal. Encoding methods，such as PCM，DPCM and ΔM，are usually used to convert a quantized signal into a binary signal. This process is lossy in the sense that some information is lost，but the loss of information is under the designer's control in that it can be made small enough.

The signal to quantization ratio of PCM system increases with the bandwidth B exponentially，while the analog modulation increases linearly. That is why PCM and its improved coding are widely used.

Homework

5.1 Suppose the spectrum of a message signal $m(t)$ is $M(f)$，its expression is

$$M(f)=\begin{cases}1-\dfrac{|f|}{200}, & |f|<200\text{Hz}\\ 0, & \text{otherwise}\end{cases}$$

(1) If the sampling rate is 300Hz, try to draw the spectrum of the sampled signal $m_s(t)$;

(2) If the sampling rate is 400Hz, try to re-draw it.

5.2 Using A-law 13 segment encoding, if the sampling value is 635, find the output of PCM encoder.

5.3 Suppose the message signal is $m(t)=9+A\cos\omega t$, where $A\leqslant 10\text{V}$. If $m(t)$ is quantized to 40 levels, try to find the number of binary bits and the quantization intervals.

Terminologies

sampling	采样	commutator	换向器
quantizing	量化	PCM	脉冲编码调制
encoding	编码	DPCM	差分脉冲编码调制
uniform quantization	均匀量化	ΔM	增量调制
nonuniform quantization	非均匀量化	PAM	脉冲幅度调制
quantization noise	量化噪声	PDM	脉冲宽度调制
equalizer	均衡器	PPM	脉冲位置调制
compressor	压缩器	TDM	时分复用