第3章

近似样条

3.1 引言

前面的章节讨论了如何使用控制点(CP)和调和函数(BF)来表征插值样条和混合样条。贝塞尔样条曲线是图形包中用于建模样条曲线的非常流行和广泛使用的工具。然而,贝塞尔样条有两个主要缺点,这一直是设计**近似样条**的动机。第一个缺点是 CP 的数量取决于曲线的阶数,即不可能在不增加多项式阶数和计算复杂度的情况下增加 CP 的数量以实现更平滑的控制。第二个缺点是没有提供**局部控制**,即移动单个 CP 的位置将改变整个样条的形状,而不是样条的局部部分。在局部图形中,控制通常是可取的,因为它可以对样条进行小幅调整[Hearn and Baker,1996]。

为了克服这些缺点,已经提出了一种称为 **B 样条**(B-spline)的新型样条。B 样条代表“基本样条”,它们是真正的近似值,即通常它们不经过任何 CP,但在特殊条件下它们可能被迫这样做。本质上,B 样条由多个在连接点处具有连续性的曲线段组成。这可以实现局部控制,即当移动 CP 时,仅更改一组局部曲线段而不是整个曲线。B 样条的 BF 是使用算法(称为 **Cox de Boor 算法**)来计算的,该算法以德国数学家 Carl-Wilhelm Reinhold de Boor 命名。连接点处的参数变量 t 的值存储在称为“**结向量**(KV)”的向量中。若结点值是等间距的,则生成的样条称为均匀 B 样条;否则,称为**非均匀 B 样条**。当 KV 值重复时,称为开放均匀 B 样条。

BF 和 B 样条方程不是单个实体,而是每个段的表达式集合。例如,若有 4 个段,分别指定为 A、B、C 和 D,则每个 BF(记为 B)具有 4 个子分量 B_A、B_B、B_C 和 B_D,并表示为 $B=\{B_A,B_B,B_C,B_D\}$。若有 3 个 CP,即 P_0、P_1 和 P_2,则将有 3 个 BF:B_0、B_1 和 B_2,每个都有 4 个子分量,即 $B_{0A},\cdots,B_{0D},B_{1A},\cdots,B_{1D}$ 和 $B_{2A},\cdots,B_{2D}$。通常每行一个 BF 表示为

$$B_0=\{B_{0A},B_{0B},B_{0C},B_{0D}\}$$

$$B_1=\{B_{1A},B_{1B},B_{1C},B_{1D}\}$$

$$B_2=\{B_{2A},B_{2B},B_{2C},B_{2D}\}$$

类似地,曲线方程 P 也将具有 4 个子分量,$P=\{P_A,P_B,P_C,P_D\}$。我们之前已经看

到，曲线方程可以表示为 CP 和 BF 的乘积，因此子分量可以写为：$P_A=P_{0.B0A}+P_{1.B1A}+P_{2.B2A}$，$P_B=P_{0.B0B}+P_{1.B1B}+P_{2.B2B}$ 等。注意，子分量对 t 的不同范围有效，因此不能相加，但需要表示为值矩阵。通常，段 A 在 $t_0\leqslant t<t_1$ 范围内有效，段 B 在 $t_1\leqslant t<t_2$ 范围内有效，以此类推。由于曲线表达式可能非常大，在大多数情况下，方程是垂直而不是水平编写的，每个段的值范围都提到了，如下所示：

$$P=\begin{cases}P_A=P_{0.B0A}+P_{1.B1A}+P_{2.B2A}(t_0\leqslant t<t_1) & [\text{段 } A]\\ P_B=P_{0.B0B}+P_{1.B1B}+P_{2.B2B}(t_1\leqslant t<t_2) & [\text{段 } B]\\ P_C=P_{0.B0C}+P_{1.B1C}+P_{2.B2C}(t_2\leqslant t<t_3) & [\text{段 } C]\end{cases}$$

以下各节将说明有关 B 和 P 值计算的详细信息。

3.2 线性均匀 B 样条

B 样条有两个定义参数：d 与样条的阶数有关，n 与 CP 的数量有关。样条的阶数实际上是$(d-1)$，CP 的数量是$(n+1)$。其他相关参数的推导如下所述。

对于线性 B 样条，我们从 $d=2$ 和 $n=2$ 开始：

曲线的阶数：$d-1=1$。

CP 数量：$n+1=3$。

BF 数量：$n+1=3$。

曲线段数：$d+n=4$。

KV 中的元素数：$d+n+1=5$。

将曲线段指定为 A、B、C 和 D，CP 为 P_0、P_1 和 P_2。将 KV 中的元素指定为$\{t_k\}$，其中 k 在值$\{0,1,2,3,4\}$上循环。在这种情况下，KV 为 $\boldsymbol{T}=[t_0,t_1,t_2,t_3,t_4]$。让 BF 以 $B_{k,d}$ 的形式指定。由于有 3 个 CP，曲线的 BF 由 $B_{0,2}$、$B_{1,2}$ 和 $B_{2,2}$ 给出，更高的 k 值在此不相关。曲线方程由下式给出：

$$P(t)=P_0B_{0,2}+P_1B_{1,2}+P_2B_{2,2} \tag{3.1}$$

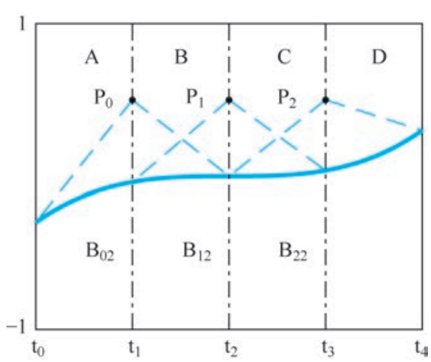

图 3.1 具有 3 个 CP 的线性均匀 B 样条

图 3.1 是一个示意图，显示了 3 个 CP，即 P_0、P_1 和 P_2 如何与 3 个 BF，即 $B_{0,2}$、$B_{1,2}$ 和 $B_{2,2}$，4 个段 A、B、C 和 D 以及五元 KV，即$[t_0,t_1,t_2,t_3,t_4]$相关。无论 CP 的实际位置如何，它们对特定段的影响保持不变。CP 中，P_0 对段 A 和 B 施加影响，P_1 对段 B 和 C 施加影响，P_2 对段 C 和 D 施加影响。这提供了 B 样条的局部控制特性：改变一个 CP 的位置只改变样条的两个段，而样条曲线的其余部分保持不变。图中虚线表示影响范围。在本节的其余部分，我们提供了对这一事实的验证。

BF 的表达式是使用 **Cox de Boor 算法**[Hearn and Baker，1996]来计算的，如下所示：

$$B_{k,1}=1,\quad \text{if}\quad t_k\leqslant t\leqslant t_{k+1},\quad \text{else } 0$$

$$B_{k,d}=\left(\frac{t-t_k}{t_{k+d-1}-t_k}\right)B_{k,d-1}+\left(\frac{t_{k+d}-t}{t_{k+d}-t_{k+1}}\right)B_{k+1,d-1} \tag{3.2}$$

第一行表明，若特定段的 t 值在特定范围内，则 BF 的 $d=1$ 的一阶项等于 1，否则为 0。第二行说明高阶项如何计算，对 $d>1$，根据一阶项计算。该算法将在后面说明。需要注意的一点是，该算法本质上是递归的，因为与 d 相关的高阶项是根据与 $d-1$ 阶相关的低阶项计算的。因此，即使曲线方程只需要有式(3.1)中所示的二阶项 $B_{k,2}$，还是首先需要计算一阶项 $B_{k,1}$。

要找到 BF 和曲线方程，必须分别分析每一段。但首先，需要 KV 的数值。让我们做一个简化的假设，KV 值如下：$t_0=0$、$t_1=1$、$t_2=2$、$t_3=3$、$t_4=4$。因此 KV 变为 $\boldsymbol{T}=[0,1,2,3,4]$。在适当时，我们将证明这个假设是正确的。

段 A：$t_0 \leqslant t < t_1$：

$$B_{0,1}=1,\quad B_{1,1}=0,\quad B_{2,1}=0,\quad B_{3,1}=0,\quad B_{4,1}=0$$

$$B_{0,2}=\left(\frac{t-t_0}{t_1-t_0}\right)B_{0,1}+\left(\frac{t_2-t}{t_2-t_1}\right)B_{1,1}=t$$

$$B_{1,2}=\left(\frac{t-t_1}{t_2-t_1}\right)B_{1,1}+\left(\frac{t_3-t}{t_3-t_2}\right)B_{2,1}=0$$

$$B_{2,2}=\left(\frac{t-t_2}{t_3-t_2}\right)B_{2,1}+\left(\frac{t_4-t}{t_4-t_3}\right)B_{3,1}=0$$

段 B：$t_1 \leqslant t < t_2$：

$$B_{0,1}=0,\quad B_{1,1}=1,\quad B_{2,1}=0,\quad B_{3,1}=0,\quad B_{4,1}=0$$

$$B_{0,2}=\left(\frac{t-t_0}{t_1-t_0}\right)B_{0,1}+\left(\frac{t_2-t}{t_2-t_1}\right)B_{1,1}=2-t$$

$$B_{1,2}=\left(\frac{t-t_1}{t_2-t_1}\right)B_{1,1}+\left(\frac{t_3-t}{t_3-t_2}\right)B_{2,1}=t-1$$

$$B_{2,2}=\left(\frac{t-t_2}{t_3-t_2}\right)B_{2,1}+\left(\frac{t_4-t}{t_4-t_3}\right)B_{3,1}=0$$

段 C：$t_2 \leqslant t < t_3$：

$$B_{0,1}=0,\quad B_{1,1}=0,\quad B_{2,1}=1,\quad B_{3,1}=0,\quad B_{4,1}=0$$

$$B_{0,2}=\left(\frac{t-t_0}{t_1-t_0}\right)B_{0,1}+\left(\frac{t_2-t}{t_2-t_1}\right)B_{1,1}=0$$

$$B_{1,2}=\left(\frac{t-t_1}{t_2-t_1}\right)B_{1,1}+\left(\frac{t_3-t}{t_3-t_2}\right)B_{2,1}=3-t$$

$$B_{2,2}=\left(\frac{t-t_2}{t_3-t_2}\right)B_{2,1}+\left(\frac{t_4-t}{t_4-t_3}\right)B_{3,1}=t-2$$

段 D：$t_3 \leqslant t < t_4$：

$$B_{0,1}=0,\quad B_{1,1}=0,\quad B_{2,1}=0,\quad B_{3,1}=1,\quad B_{4,1}=0$$

$$B_{0,2}=\left(\frac{t-t_0}{t_1-t_0}\right)B_{0,1}+\left(\frac{t_2-t}{t_2-t_1}\right)B_{1,1}=0$$

$$B_{1,2}=\left(\frac{t-t_1}{t_2-t_1}\right)B_{1,1}+\left(\frac{t_3-t}{t_3-t_2}\right)B_{2,1}=0$$

$$B_{2,2}=\left(\frac{t-t_2}{t_3-t_2}\right)B_{2,1}+\left(\frac{t_4-t}{t_4-t_3}\right)B_{3,1}=4-t$$

注意，每段都有一个特定的 k 值，但我们在计算 BF 时仍然在其他可能的值上循环 k，因为我们想找出其他 CP 对那个段的影响。例如，对于段 A，$B_{0,2}=0$，但是我们计算 $B_{1,2}$ 和 $B_{2,2}$ 来找出第二个 CP 和第三个 CP 对段 A 的影响。在这种情况下，我们看到 $B_{1,2}$ 和 $B_{2,2}$ 为零，表示段 A 仅受第一个 CP 即 P_0 控制，不受其他 CP 即 P_1 和 P_2 影响，见式(3.1)。同样对于段 B，$k=1$，但我们看到 $B_{0,2}$ 和 $B_{1,2}$ 都是非零值，这意味着段 B 受两个 CP，即 P_0 和 P_1 的影响。

现在根据式(3.1)，确定曲线公式所需的 BF 为 $B_{0,2}$、$B_{1,2}$ 和 $B_{3,2}$。然而，这些 BF 对于不同的段具有不同的值。因此，需要将所有这些值收集在一起，指定它们有效的段。这里使用单独的下标 A、B、C 和 D 来表示相关段。

$$\begin{cases}B_{0,2}=\{B_{0,2A},B_{0,2B},B_{0,2C},B_{0,2D}\}\\B_{1,2}=\{B_{1,2A},B_{1,2B},B_{1,2C},B_{1,2D}\}\\B_{2,2}=\{B_{2,2A},B_{2,2B},B_{2,2C},B_{2,2D}\}\end{cases}\tag{3.3}$$

表 3.1 总结了 BF 值的分段计算。如前所述，假设 KV 为 $\boldsymbol{T}=[0,1,2,3,4]$。

表 3.1 线性均匀 B 样条的 BF 计算

段	t	$B_{k,1}$	$B_{k,2}$
A	$t_0\leqslant t<t_1$	$B_{0,1}=1$	$B_{0,2}=t$
		$B_{1,1}=0$	$B_{1,2}=0$
		$B_{2,1}=0$	$B_{2,2}=0$
		$B_{3,1}=0$	
B	$t_1\leqslant t<t_2$	$B_{0,1}=0$	$B_{0,2}=2-t$
		$B_{1,1}=1$	$B_{1,2}=t-1$
		$B_{2,1}=0$	$B_{2,2}=0$
		$B_{3,1}=0$	
C	$t_2\leqslant t<t_3$	$B_{0,1}=0$	$B_{0,2}=0$
		$B_{1,1}=0$	$B_{1,2}=3-t$
		$B_{2,1}=1$	$B_{2,2}=t-2$
		$B_{3,1}=0$	
D	$t_3\leqslant t<t_4$	$B_{0,1}=0$	$B_{0,2}=0$
		$B_{1,1}=0$	$B_{1,2}=0$
		$B_{2,1}=0$	$B_{2,2}=4-t$
		$B_{3,1}=1$	

将上述值代入式(3.3)，得到：

$$\begin{cases}B_{0,2}=\{t,2-t,0,0\}\\B_{1,2}=\{0,t-1,3-t,0\}\\B_{2,2}=\{0,0,t-2,4-t\}\end{cases}\tag{3.4}$$

式(3.4)指定了具有 3 个 CP(由 3 个垂直行表示)和 4 个 CP(由每行 4 个元素向量表示)的线性均匀 B 样条的 BF。

式(3.5)显示了 BF 的另一种表示方式，其中仅指示非零值以及它们有效的段名称和 t 的范围。

$$\begin{cases} B_{0,2}=\begin{cases} t & (0 \leqslant t < 1)A \\ 2-t & (1 \leqslant t < 2)B \end{cases} \\ B_{1,2}=\begin{cases} t-1 & (1 \leqslant t < 2)B \\ 3-t & (2 \leqslant t < 3)C \end{cases} \\ B_{2,2}=\begin{cases} t-2 & (2 \leqslant t < 3)C \\ 4-t & (3 \leqslant t < 4)D \end{cases} \end{cases} \tag{3.5}$$

式(3.5)表明 $B_{0,2}$ 并因此和 P_0 影响分段 A 和 B，$B_{1,2}$ 和 P_1 影响分段 B 和 C，而 $B_{2,2}$ 和 P_2 影响分段 C 和 D，这一事实已由图 3.1 中的虚线指出。

BF 的图如图 3.2 所示。每个 BF 具有相同的形状，但相对于前一个向右移动 1。因此，每个 BF 都可以通过将 t 替换为$(t-1)$从前一个 BF 中获得，如式(3.5)所示。同样如式(3.4)所示，每个 BF 有四个细分，其中两个是非零的。$B_{0,2}$ 的第一条曲线对于分段 $A(0 \leqslant t<1)$ 和 $B(1 \leqslant t<2)$具有非零部分；$B_{1,2}$ 的第二条曲线对于分段 $B(1 \leqslant t<2)$和 $C(2 \leqslant t<3)$，$B_{2,2}$ 的第三条曲线对于段 $C(2 \leqslant t<3)$和 $D(3 \leqslant t<4)$也具有非零部分。由于 BF 与 CP 相关联，这再次提供了对样条局部控制属性的指示，即第一个 CP 对前两个段 A 和 B 有影响，第二个 CP 对 B 和 C 有影响，以此类推。这意味着如果第一个 CP 改变，它将只影响 4 条线段中的两条，而样条线的其余部分将保持不变。这与每个 BF 在整个 t 范围内有效的插值曲线和混合曲线形成对比。

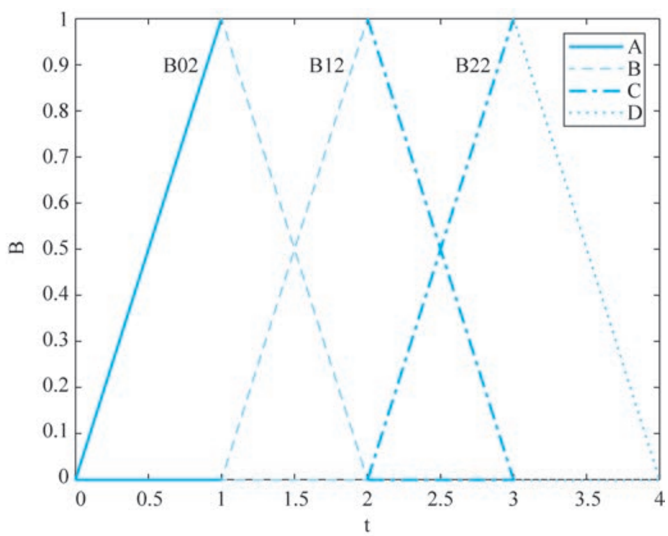

图 3.2 线性均匀 B 样条的 BF

样条方程是其四段方程的集合：

$$P(t)=\begin{cases} P_A & (0 \leqslant t < 1) \\ P_B & (1 \leqslant t < 2) \\ P_C & (2 \leqslant t < 3) \\ P_D & (3 \leqslant t < 4) \end{cases} \tag{3.6}$$

其中，

$$\begin{cases} P_A = P_0 B_{0,2A} + P_1 B_{1,2A} + P_2 B_{2,2A} \\ P_B = P_0 B_{0,2B} + P_1 B_{1,2B} + P_2 B_{2,2B} \\ P_C = P_0 B_{0,2C} + P_1 B_{1,2C} + P_2 B_{2,2C} \\ P_D = P_0 B_{0,2D} + P_1 B_{1,2D} + P_2 B_{2,2D} \end{cases} \tag{3.7}$$

将表 3.1 中的 BF 值代入式(3.6),可以得到:

$$P(t) = \begin{cases} P_0 t & (0 \leqslant t < 1) \\ P_0(2-t) + P_1(t-1) & (1 \leqslant t < 2) \\ P_1(3-t) + P_2(t-2) & (2 \leqslant t < 3) \\ P_2(4-t) & (3 \leqslant t < 4) \end{cases} \tag{3.8}$$

式(3.8)表示具有 3 个 CP 和 4 个段的线性均匀 B 样条方程。这 4 部分是 4 个段的方程。

例 3.1 找到具有 CP(2,−3)、(5,5)和(8,−1)的线性均匀 B 样条的方程。还要编写一个程序来绘制 BF。

解:

根据式(3.8),代入给定 CP 的值:

$$x(t) = \begin{cases} 2t & (0 \leqslant t < 1) \\ 3t - 1 & (1 \leqslant t < 2) \\ 3t - 1 & (2 \leqslant t < 3) \\ -8t + 32 & (3 \leqslant t < 4) \end{cases}$$

$$y(t) = \begin{cases} -3t & (0 \leqslant t < 1) \\ 8t - 11 & (1 \leqslant t < 2) \\ -6t + 17 & (2 \leqslant t < 3) \\ t - 4 & (3 \leqslant t < 4) \end{cases}$$

MATLAB Code 3.1

```
clear all; format compact; clc;
t0 = 0; t1 = 1; t2 = 2; t3 = 3; t4 = 4;
T = [t0, t1, t2, t3, t4];
syms t P0 P1 P2;

% Segment A
B01 = 1; B11 = 0; B21 = 0; B31 = 0; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02A = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12A = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22A = B22;

% Segment B
B01 = 0; B11 = 1; B21 = 0; B31 = 0; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02B = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12B = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22B = B22;

% Segment C
B01 = 0; B11 = 0; B21 = 1; B31 = 0; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02C = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12C = B12;
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S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22C = B22;

% Segment D
B01 = 0; B11 = 0; B21 = 0; B31 = 1; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02D = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12D = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22D = B22;
fprintf('Blending functions :\n');
B02 = [B02A, B02B, B02C, B02D]
B12 = [B12A, B12B, B12C, B12D]
B22 = [B22A, B22B, B22C, B22D]
fprintf('\n');
fprintf('General Equation of Curve :\n');
P = P0 * B02 + P1 * B12 + P2 * B22
fprintf('\n');
x0 = 2; x1 = 5; x2 = 8;
y0 = -3; y1 = 5; y2 = -1;
fprintf('Actual Equation :\n');
x = subs(P, ([P0, P1, P2]), ([x0, x1, x2]))
y = subs(P, ([P0, P1, P2]), ([y0, y1, y2]))

% plotting BFs
tta = linspace(t0, t1);
ttb = linspace(t1, t2);
ttc = linspace(t2, t3);
ttd = linspace(t3, t4);
B02aa = subs(B02A, t, tta);
B02bb = subs(B02B, t, ttb);
B02cc = subs(B02C, t, ttc);
B02dd = subs(B02D, t, ttd);
B12aa = subs(B12A, t, tta);
B12bb = subs(B12B, t, ttb);
B12cc = subs(B12C, t, ttc);
B12dd = subs(B12D, t, ttd);
B22aa = subs(B22A, t, tta);
B22bb = subs(B22B, t, ttb);
B22cc = subs(B22C, t, ttc);
B22dd = subs(B22D, t, ttd);
figure
plot(tta, B02aa, 'k-', ttb,B02bb, 'k--', ttc,B02cc, 'k-.', ttd,B02dd, 'k:'); hold on;
plot(tta, B12aa, 'k-', ttb,B12bb, 'k--', ttc,B12cc, 'k-.', ttd,B12dd, 'k:');
plot(tta, B22aa, 'k-', ttb,B22bb, 'k--', ttc,B22cc, 'k-.', ttd,B22dd, 'k:'); hold off;
xlabel('t'); ylabel('B');
legend('A', 'B', 'C', 'D');
text(0.6, 0.9, 'B02'); text(1.6, 0.9, 'B12'); text(2.6, 0.9, 'B22');
```

注解

有关 BF 的图，请参见图 3.2。

3.3　改变控制点的数量

设计 **B 样条**的目标之一是进行局部控制，我们在 3.2 节中已经看到这一事实是合理的，因为不同的 CP 只影响特定的曲线段而不是整个曲线。另一个目标是使 CP 的数量与曲线

的阶数无关。为了研究这一点，让我们现在将 CP 的数量增加 1 个，同时保持与以前相同的阶次，并找出这样的组合是否会产生有效的曲线方程。

对于这种情况，我们从 $d=2$ 和 $n=3$ 开始：

曲线的阶数：$d-1=1$。

CP 数量：$n+1=4$。

曲线段数：$d+n=5$。

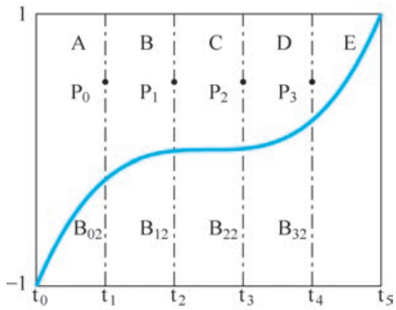

图 3.3　具有 4 个 CP 的线性均匀 B 样条

将曲线段指定为 A、B、C、D 和 E（见图 3.3），CP 为 P_0、P_1、P_2 和 P_3。

KV 中的元素数量：$d+n+1=6$。

将 KV 中的元素指定为 $T=\{t_k\}$，其中 k 在值$\{0,1,2,3,4,5\}$上循环。在这种情况下，$\boldsymbol{T}=[t_0,t_1,t_2,t_3,t_4,t_5]$。

BF 的数量与 CP 的数量相同，即 4。将 BF 指定为 $B_{k,d}$。在这种情况下，BF 是 $B_{0,2}$、$B_{1,2}$、$B_{2,2}$ 和 $B_{3,2}$。较高的 k 值无关紧要，因此不需要计算。曲线方程由下式给出：

$$P(t)=P_0B_{0,2}+P_1B_{1,2}+P_2B_{2,2}+P_3B_{3,2} \tag{3.9}$$

如果遵循 3.2 节中概述的过程，读者将能够验证这确实产生了有效的 B 样条。因此，它被留作练习。下面给出最终的 BF 和曲线方程作为参考。与式(3.5)和式(3.8)进行比较以查看差异。

$$B_{0,2}=\begin{cases} t & (0\leqslant t<1) \\ 2-t & (1\leqslant t<2) \end{cases}$$

$$B_{1,2}=\begin{cases} t-1 & (1\leqslant t<2) \\ 3-t & (2\leqslant t<3) \end{cases}$$

$$B_{2,2}=\begin{cases} t-2 & (2\leqslant t<3) \\ 4-t & (3\leqslant t<4) \end{cases}$$

$$B_{3,2}=\begin{cases} t-3 & (3\leqslant t<4) \\ 5-t & (4\leqslant t<5) \end{cases}$$

$$P(t)=\begin{cases} P_0t & (0\leqslant t<1) \\ P_0(2-t)+P_1(t-1) & (1\leqslant t<2) \\ P_1(3-t)+P_2(t-2) & (2\leqslant t<3) \\ P_2(4-t)+P_3(t-3) & (3\leqslant t<4) \\ P_3(5-t) & (4\leqslant t<5) \end{cases}$$

3.4　二次均匀 B 样条

为了生成二次 B 样条，我们从 $d=3$ 和 $n=3$ 开始：

曲线的阶数：$d-1=2$。

CP 数量：$n+1=4$。

BF 数量：$n+1=4$。

曲线段数：$d+n=6$。

KV 中的元素数：$d+n+1=7$。

设曲线段为 A、B、C、D、E 和 F，CP 为 P_0、P_1、P_2 和 P_3（见图 3.4）。

对于 $k=\{0,1,2,3,4,5,6\}$，令 KV 为 $T=\{t_k\}$。在这种情况下，$\boldsymbol{T}=[t_0,t_1,t_2,t_3,t_4,t_5,t_6]$。

设 BF 为 $B_{0,3}$、$B_{1,3}$、$B_{2,3}$ 和 $B_{3,3}$。

曲线方程为

$$P(t)=P_0B_{0,3}+P_1B_{1,3}+P_2B_{2,3}+P_3B_{3,3} \tag{3.10}$$

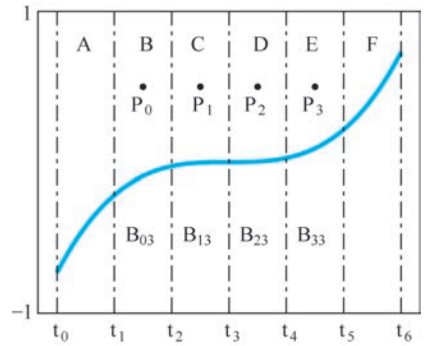

图 3.4　具有 4 个 CP 的二次均匀 B 样条

如前所述，我们假设 KV 为 $\boldsymbol{T}=[0,1,2,3,4,5,6]$。根据 **Cox de Boor 算法**的第一个条件，一阶项 $B_{0,3}$、$B_{0,1}$、$B_{1,1}$、$B_{2,1}$、$B_{3,1}$ 和 $B_{4,1}$ 将为 0 或 1。根据算法的第二个条件，二阶项计算如下：

$$\begin{cases} B_{0,2}=(t-0)B_{0,1}+(2-t)B_{1,1} \\ B_{1,2}=(t-1)B_{1,1}+(3-t)B_{2,1} \\ B_{2,2}=(t-2)B_{2,1}+(4-t)B_{3,1} \\ B_{3,2}=(t-3)B_{3,1}+(5-t)B_{4,1} \\ B_{4,2}=(t-4)B_{4,1}+(6-t)B_{5,1} \end{cases} \tag{3.11}$$

这里也有三阶项，可由二阶项计算得出：

$$\begin{cases} B_{0,3}=\left(\frac{1}{2}\right)(t-0)B_{0,2}+\left(\frac{1}{2}\right)(3-t)B_{1,2} \\ B_{1,3}=\left(\frac{1}{2}\right)(t-1)B_{1,2}+\left(\frac{1}{2}\right)(4-t)B_{2,2} \\ B_{2,3}=\left(\frac{1}{2}\right)(t-2)B_{2,2}+\left(\frac{1}{2}\right)(5-t)B_{3,2} \\ B_{3,3}=\left(\frac{1}{2}\right)(t-3)B_{3,2}+\left(\frac{1}{2}\right)(6-t)B_{4,2} \end{cases} \tag{3.12}$$

由于有 6 部分，每个 BF 包含 6 个子分量：

$$\begin{cases} B_{0,3}=\{B_{0,3A},B_{0,3B},B_{0,3C},B_{0,3D},B_{0,3E},B_{0,3F}\} \\ B_{1,3}=\{B_{1,3A},B_{1,3B},B_{1,3C},B_{1,3D},B_{1,3E},B_{1,3F}\} \\ B_{2,3}=\{B_{2,3A},B_{2,3B},B_{2,3C},B_{2,3D},B_{2,3E},B_{2,3F}\} \\ B_{3,3}=\{B_{3,3A},B_{3,3B},B_{3,3C},B_{3,3D},B_{3,3E},B_{3,3F}\} \end{cases} \tag{3.13}$$

表 3.2 总结了 BF 值的计算。

表 3.2　二次均匀 B 样条的 BF 计算

段	t	$\boldsymbol{B}_{k,1}$	$\boldsymbol{B}_{k,2}$	$\boldsymbol{B}_{k,3}$
A	$0\leqslant t<1$	$B_{0,1}=1$	$B_{0,2}=t$	$B_{0,3}=(1/2)t^2$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$

续表

段	t	$B_{k,1}$	$B_{k,2}$	$B_{k,3}$
A	$0\leqslant t<1$	$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
B	$1\leqslant t<2$	$B_{0,1}=0$	$B_{0,2}=2-t$	$B_{0,3}=-t^2+3t-(3/2)$
		$B_{1,1}=1$	$B_{1,2}=t-1$	$B_{1,3}=(1/2)(t-1)^2$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
C	$2\leqslant t<3$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=(1/2)(t-3)^2$
		$B_{1,1}=0$	$B_{1,2}=3-t$	$B_{1,3}=-t^2+5t-(11/2)$
		$B_{2,1}=1$	$B_{2,2}=t-2$	$B_{2,3}=(1/2)(t-2)^2$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
D	$3\leqslant t<4$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=(1/2)(t-4)^2$
		$B_{2,1}=0$	$B_{2,2}=4-t$	$B_{2,3}=-t^2+7t-(23/2)$
		$B_{3,1}=1$	$B_{3,2}=t-3$	$B_{3,3}=(1/2)(t-3)^2$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
E	$4\leqslant t<5$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=(1/2)(t-5)^2$
		$B_{3,1}=0$	$B_{3,2}=5-t$	$B_{3,3}=-t^2+9t-(39/2)$
		$B_{4,1}=1$	$B_{4,2}=t-4$	
		$B_{5,1}=0$		
F	$5\leqslant t<6$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=(1/2)(t-6)^2$
		$B_{4,1}=0$	$B_{4,2}=6-t$	
		$B_{5,1}=1$		

将上述值代入式(3.13)，得到：

$$B_{0,3}=\begin{cases}(1/2)t^2 & (0\leqslant t<1)A\\ -t^2+3t-(3/2) & (1\leqslant t<2)B\\ (1/2)(t-3)^2 & (2\leqslant t<3)C\end{cases}$$

$$B_{1,3}=\begin{cases}(1/2)(t-1)^2 & (1\leqslant t<2)B\\ -t^2+5t-(11/2) & (2\leqslant t<3)C\\ (1/2)(t-4)^2 & (3\leqslant t<4)D\end{cases}$$

$$B_{2,3}=\begin{cases}(1/2)(t-2)^2 & (2\leqslant t<3)C\\ -t^2+7t-(23/2) & (3\leqslant t<4)D\\ (1/2)(t-5)^2 & (4\leqslant t<5)E\end{cases} \quad (3.14)$$

$$B_{3,3}=\begin{cases}(1/2)(t-3)^2 & (3\leqslant t<4)D\\ -t^2+9t-(39/2) & (4\leqslant t<5)E\\ (1/2)(t-6)^2 & (5\leqslant t<6)F\end{cases}$$

式(3.14)表示具有 4 个 CP 和 6 个段的二次均匀 B 样条的 BF。BF 的图如图 3.5 所示。每个 BF 具有相同的形状,但相对于前一个 BF 向右移动 1。因此,每个 BF 可以通过将 t 替换为$(t-1)$从前一个 BF 中获得。每个 BF 有 6 个细分,其中 3 个是非零的。$B_{0,3}$ 的第一条曲线对于段 $A(0\leqslant t<1)$、$B(1\leqslant t<2)$、$C(2\leqslant t<3)$具有非零部分,$B_{1,3}$ 的第二条曲线对于段 $B(1\leqslant t<2)$、$C(2\leqslant t<3)$、$D(3\leqslant t<4)$具有非零部分,$B_{2,3}$ 的第三条曲线对于段 $C(2\leqslant t<3)$、$D(3\leqslant t<4)$、$E(4\leqslant t<5)$具有非零部分,$B_{3,3}$ 的第四条曲线对于段 $D(3\leqslant t<4)$、$E(4\leqslant t<5)$、$F(5\leqslant t<6)$具有非零部分。

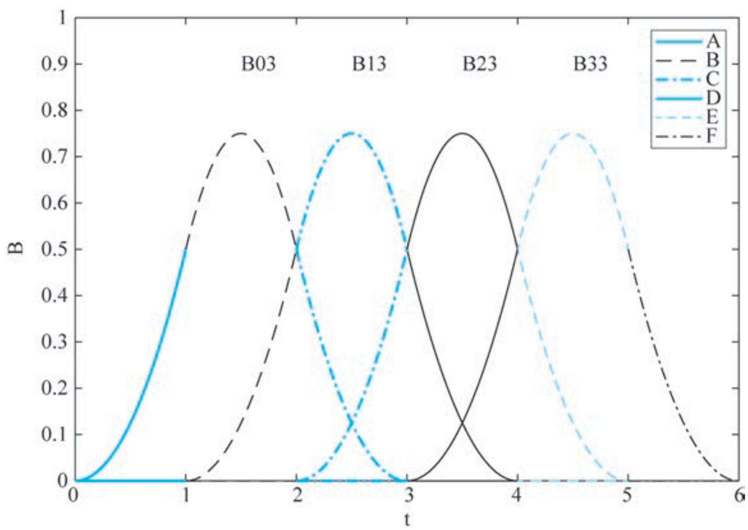

图 3.5 二次均匀 B 样条的 BF

样条方程是其六段方程的集合:

$$P(t)=\begin{cases}P_A & (0\leqslant t<1)\\ P_B & (1\leqslant t<2)\\ P_C & (2\leqslant t<3)\\ P_D & (3\leqslant t<4)\\ P_E & (4\leqslant t<5)\\ P_F & (5\leqslant t<6)\end{cases} \quad (3.15)$$

其中，

$$\begin{cases} P_A = P_0 B_{0,3A} + P_1 B_{1,3A} + P_2 B_{2,3A} + P_3 B_{3,3A} \\ P_B = P_0 B_{0,3B} + P_1 B_{1,3B} + P_2 B_{2,3B} + P_3 B_{3,3B} \\ P_C = P_0 B_{0,3C} + P_1 B_{1,3C} + P_2 B_{2,3C} + P_3 B_{3,3C} \\ P_D = P_0 B_{0,3D} + P_1 B_{1,3D} + P_2 B_{2,3D} + P_3 B_{3,3D} \\ P_E = P_0 B_{0,3E} + P_1 B_{1,3E} + P_2 B_{2,3E} + P_3 B_{3,3E} \\ P_F = P_0 B_{0,3F} + P_1 B_{1,3F} + P_2 B_{2,3F} + P_3 B_{3,3F} \end{cases} \tag{3.16}$$

将表 3.2 中的 BF 值代入式(3.15)并指定它们的有效范围，可以得到：

$$P(t) = \begin{cases} P_0(1/2)t^2 & (0 \leqslant t < 1) \\ P_0(-t^2+3t-3/2) + P_1(1/2)(t-1)^2 & (1 \leqslant t < 2) \\ P_0(1/2)(t-3)^2 + P_1(-t^2+5t-11/2) + P_2(1/2)(t-2)^2 & (2 \leqslant t < 3) \\ P_1(1/2)(t-4)^2 + P_2(-t^2+7t-23/2) + P_3(1/2)(t-3)^2 & (3 \leqslant t < 4) \\ P_2(1/2)(t-5)^2 + P_3(-t^2+9t-39/2) & (4 \leqslant t < 5) \\ P_3(1/2)(t-6)^2 & (5 \leqslant t < 6) \end{cases} \tag{3.17}$$

式(3.17)表示具有 4 个 CP 和 6 个段的二次均匀 B 样条方程。方程的 6 部分代表曲线的 6 个段。

例 3.2 求具有 CP，即 $P_0(1,2)$、$P_1(4,1)$、$P_2(6,5)$和 $P_3(8,-1)$的均匀二次 B 样条方程。还要编写一个程序来绘制 BF 和实际曲线。

解：

根据式(3.17)，代入给定 CP 的值(见图 3.6)：

$$x(t) = \begin{cases} t^2/2 & (0 \leqslant t < 1) \\ t^2 - t + 1/2 & (1 \leqslant t < 2) \\ -t^2/2 + 5t - 11/2 & (2 \leqslant t < 3) \\ 2t - 1 & (3 \leqslant t < 4) \\ -5t^2 + 42t - 81 & (4 \leqslant t < 5) \\ 4(t-6)^2 & (5 \leqslant t < 6) \end{cases}$$

$$y(t) = \begin{cases} t^2 & (0 \leqslant t < 1) \\ -3t^2/2 + 5t - 5/2 & (1 \leqslant t < 2) \\ 5t^2/2 - 11t + 27/2 & (2 \leqslant t < 3) \\ -5t^2 + 34t - 54 & (3 \leqslant t < 4) \\ 7t^2/2 - 34t + 82 & (4 \leqslant t < 5) \\ -(1/2)(t-6)^2 & (5 \leqslant t < 6) \end{cases}$$

MATLAB Code 3.2

```
clear all; format compact; clc;
t0 = 0; t1 = 1; t2 = 2; t3 = 3; t4 = 4; t5 = 5; t6 = 6;
```

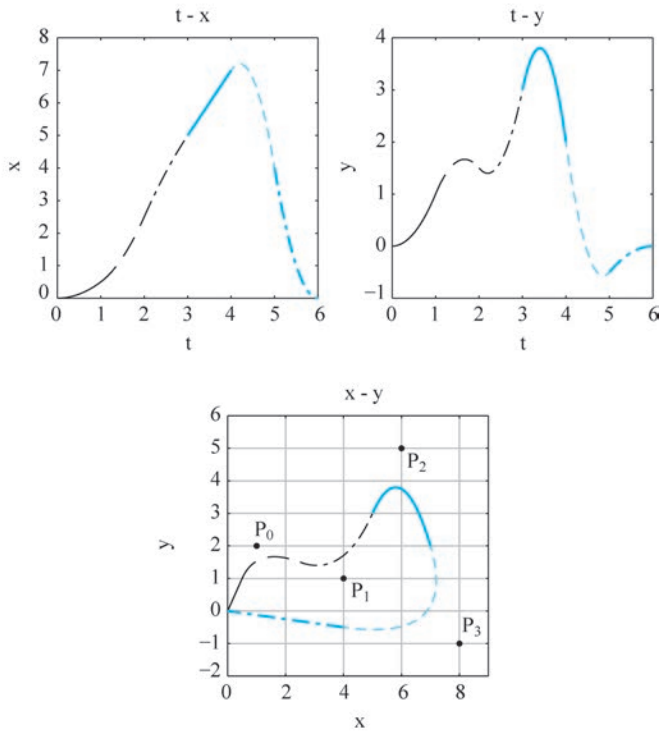

图 3.6 例 3.2 的绘图

```
T = [t0, t1, t2, t3, t4, t5, t6];
syms t P0 P1 P2 P3;

% Segment A
B01 = 1; B11 = 0; B21 = 0; B31 = 0; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02A = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12A = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22A = B22;
S1 = (t - t3)/(t4 - t3); S2 = (t5 - t)/(t5 - t4); B32 = S1 * B31 + S2 * B41; B32A = B32;
S1 = (t - t4)/(t5 - t4); S2 = (t6 - t)/(t6 - t5); B42 = S1 * B41 + S2 * B51; B42A = B42;
S1 = (t - t0)/(t2 - t0); S2 = (t3 - t)/(t3 - t1); B03 = S1 * B02 + S2 * B12; B03A = B03;
S1 = (t - t1)/(t3 - t1); S2 = (t4 - t)/(t4 - t2); B13 = S1 * B12 + S2 * B22; B13A = B13;
S1 = (t - t2)/(t4 - t2); S2 = (t5 - t)/(t5 - t3); B23 = S1 * B22 + S2 * B32; B23A = B23;
S1 = (t - t3)/(t5 - t3); S2 = (t6 - t)/(t6 - t4); B33 = S1 * B32 + S2 * B42; B33A = B33;

% Segment B
B01 = 0; B11 = 1; B21 = 0; B31 = 0; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02B = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12B = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22B = B22;
S1 = (t - t3)/(t4 - t3); S2 = (t5 - t)/(t5 - t4); B32 = S1 * B31 + S2 * B41; B32B = B32;
S1 = (t - t4)/(t5 - t4); S2 = (t6 - t)/(t6 - t5); B42 = S1 * B41 + S2 * B51; B42B = B42;
S1 = (t - t0)/(t2 - t0); S2 = (t3 - t)/(t3 - t1); B03 = S1 * B02 + S2 * B12; B03B = B03;
S1 = (t - t1)/(t3 - t1); S2 = (t4 - t)/(t4 - t2); B13 = S1 * B12 + S2 * B22; B13B = B13;
S1 = (t - t2)/(t4 - t2); S2 = (t5 - t)/(t5 - t3); B23 = S1 * B22 + S2 * B32; B23B = B23;
S1 = (t - t3)/(t5 - t3); S2 = (t6 - t)/(t6 - t4); B33 = S1 * B32 + S2 * B42; B33B = B33;
```

```
% Segment C
B01 = 0; B11 = 0; B21 = 1; B31 = 0; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02C = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12C = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22C = B22;
S1 = (t - t3)/(t4 - t3); S2 = (t5 - t)/(t5 - t4); B32 = S1 * B31 + S2 * B41; B32C = B32;
S1 = (t - t4)/(t5 - t4); S2 = (t6 - t)/(t6 - t5); B42 = S1 * B41 + S2 * B51; B42C = B42;
S1 = (t - t0)/(t2 - t0); S2 = (t3 - t)/(t3 - t1); B03 = S1 * B02 + S2 * B12; B03C = B03;
S1 = (t - t1)/(t3 - t1); S2 = (t4 - t)/(t4 - t2); B13 = S1 * B12 + S2 * B22; B13C = B13;
S1 = (t - t2)/(t4 - t2); S2 = (t5 - t)/(t5 - t3); B23 = S1 * B22 + S2 * B32; B23C = B23;
S1 = (t - t3)/(t5 - t3); S2 = (t6 - t)/(t6 - t4); B33 = S1 * B32 + S2 * B42; B33C = B33;

% Segment D
B01 = 0; B11 = 0; B21 = 0; B31 = 1; B41 = 0; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02D = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12D = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22D = B22;
S1 = (t - t3)/(t4 - t3); S2 = (t5 - t)/(t5 - t4); B32 = S1 * B31 + S2 * B41; B32D = B32;
S1 = (t - t4)/(t5 - t4); S2 = (t6 - t)/(t6 - t5); B42 = S1 * B41 + S2 * B51; B42D = B42;
S1 = (t - t0)/(t2 - t0); S2 = (t3 - t)/(t3 - t1); B03 = S1 * B02 + S2 * B12; B03D = B03;
S1 = (t - t1)/(t3 - t1); S2 = (t4 - t)/(t4 - t2); B13 = S1 * B12 + S2 * B22; B13D = B13;
S1 = (t - t2)/(t4 - t2); S2 = (t5 - t)/(t5 - t3); B23 = S1 * B22 + S2 * B32; B23D = B23;
S1 = (t - t3)/(t5 - t3); S2 = (t6 - t)/(t6 - t4); B33 = S1 * B32 + S2 * B42; B33D = B33;

% Segment E
B01 = 0; B11 = 0; B21 = 0; B31 = 0; B41 = 1; B51 = 0; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02E = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12E = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22E = B22;
S1 = (t - t3)/(t4 - t3); S2 = (t5 - t)/(t5 - t4); B32 = S1 * B31 + S2 * B41; B32E = B32;
S1 = (t - t4)/(t5 - t4); S2 = (t6 - t)/(t6 - t5); B42 = S1 * B41 + S2 * B51; B42E = B42;
S1 = (t - t0)/(t2 - t0); S2 = (t3 - t)/(t3 - t1); B03 = S1 * B02 + S2 * B12; B03E = B03;
S1 = (t - t1)/(t3 - t1); S2 = (t4 - t)/(t4 - t2); B13 = S1 * B12 + S2 * B22; B13E = B13;
S1 = (t - t2)/(t4 - t2); S2 = (t5 - t)/(t5 - t3); B23 = S1 * B22 + S2 * B32; B23E = B23;
S1 = (t - t3)/(t5 - t3); S2 = (t6 - t)/(t6 - t4); B33 = S1 * B32 + S2 * B42; B33E = B33;

% Segment F
B01 = 0; B11 = 0; B21 = 0; B31 = 0; B41 = 0; B51 = 1; B61 = 0;
S1 = (t - t0)/(t1 - t0); S2 = (t2 - t)/(t2 - t1); B02 = S1 * B01 + S2 * B11; B02F = B02;
S1 = (t - t1)/(t2 - t1); S2 = (t3 - t)/(t3 - t2); B12 = S1 * B11 + S2 * B21; B12F = B12;
S1 = (t - t2)/(t3 - t2); S2 = (t4 - t)/(t4 - t3); B22 = S1 * B21 + S2 * B31; B22F = B22;
S1 = (t - t3)/(t4 - t3); S2 = (t5 - t)/(t5 - t4); B32 = S1 * B31 + S2 * B41; B32F = B32;
S1 = (t - t4)/(t5 - t4); S2 = (t6 - t)/(t6 - t5); B42 = S1 * B41 + S2 * B51; B42F = B42;
S1 = (t - t0)/(t2 - t0); S2 = (t3 - t)/(t3 - t1); B03 = S1 * B02 + S2 * B12; B03F = B03;
S1 = (t - t1)/(t3 - t1); S2 = (t4 - t)/(t4 - t2); B13 = S1 * B12 + S2 * B22; B13F = B13;
S1 = (t - t2)/(t4 - t2); S2 = (t5 - t)/(t5 - t3); B23 = S1 * B22 + S2 * B32; B23F = B23;
S1 = (t - t3)/(t5 - t3); S2 = (t6 - t)/(t6 - t4); B33 = S1 * B32 + S2 * B42; B33F = B33;
fprintf('Blending functions :\n');
B03 = [B03A, B03B, B03C, B03D, B03E, B03F]; B03 = simplify(B03)
B13 = [B13A, B13B, B13C, B13D, B13E, B13F]; B13 = simplify(B13)
B23 = [B23A, B23B, B23C, B23D, B23E, B23F]; B23 = simplify(B23)
B33 = [B33A, B33B, B33C, B33D, B33E, B33F]; B33 = simplify(B33)
fprintf('\n');
fprintf('General Equation of Curve :\n');
```

```
P = P0 * B03 + P1 * B13 + P2 * B23 + P3 * B33
fprintf('\n');
x0 = 1; x1 = 4; x2 = 6; x3 = 8;
y0 = 2; y1 = 1; y2 = 5; y3 = -1;
fprintf('Actual Equation :\n');
y = subs(P, ([P0, P1, P2, P3]), ([y0, y1, y2, y3])); y = simplify(y)

%plotting BF
tta = linspace(t0, t1);
ttb = linspace(t1, t2);
ttc = linspace(t2, t3);
ttd = linspace(t3, t4);
tte = linspace(t4, t5);
ttf = linspace(t5, t6);
B03aa = subs(B03A, t, tta);
B03bb = subs(B03B, t, ttb);
B03cc = subs(B03C, t, ttc);
B03dd = subs(B03D, t, ttd);
B03ee = subs(B03E, t, tte);
B03ff = subs(B03F, t, ttf);
B13aa = subs(B13A, t, tta);
B13bb = subs(B13B, t, ttb);
B13cc = subs(B13C, t, ttc);
B13dd = subs(B13D, t, ttd);
B13ee = subs(B13E, t, tte);
B13ff = subs(B13F, t, ttf);
B23aa = subs(B23A, t, tta);
B23bb = subs(B23B, t, ttb);
B23cc = subs(B23C, t, ttc);
B23dd = subs(B23D, t, ttd);
B23ee = subs(B23E, t, tte);
B23ff = subs(B23F, t, ttf);
B33aa = subs(B33A, t, tta);
B33bb = subs(B33B, t, ttb);
B33cc = subs(B33C, t, ttc);
B33dd = subs(B33D, t, ttd);
B33ee = subs(B33E, t, tte);
B33ff = subs(B33F, t, ttf);
figure,
plot(tta, B03aa, 'k-', ttb, B03bb, 'k--', ttc, B03cc, 'k-.', ...
  ttd, B03dd, 'b-', tte, B03ee, 'b--', ttf, B03ff, 'b-.');
hold on;
plot(tta, B13aa, 'k-', ttb, B13bb, 'k--', ttc, B13cc, 'k-.', ...
  ttd, B13dd, 'b-', tte, B13ee, 'b--', ttf, B13ff, 'b-.');
plot(tta, B23aa, 'k-', ttb, B23bb, 'k--', ttc, B23cc, 'k-.', ...
  ttd, B23dd, 'b-', tte, B23ee, 'b--', ttf, B23ff, 'b-.');
plot(tta, B33aa, 'k-', ttb, B33bb, 'k--', ttc, B33cc, 'k-.', ...
  ttd, B33dd, 'b-', tte, B33ee, 'b--', ttf, B33ff, 'b-.');
xlabel ('t'); ylabel('B'); title('B03 - B13 - B23 - B33');
legend('A', 'B', 'C', 'D', 'E', 'F');
hold off;
```

```
% plotting curve
xa = x(1); ya = y(1);
xb = x(2); yb = y(2);
xc = x(3); yc = y(3);
xd = x(4); yd = y(4);
xe = x(5); ye = y(5);
xf = x(6); yf = y(6);
xaa = subs(xa, t, tta); yaa = subs(ya, t, tta);
xbb = subs(xb, t, ttb); ybb = subs(yb, t, ttb);
xcc = subs(xc, t, ttc); ycc = subs(yc, t, ttc);
xdd = subs(xd, t, ttd); ydd = subs(yd, t, ttd);
xee = subs(xe, t, tte); yee = subs(ye, t, tte);
xff = subs(xf, t, ttf); yff = subs(yf, t, ttf);
X = [x0, x1, x2, x3]; Y = [y0, y1, y2, y3];
figure
subplot(131),
plot(tta, xaa, 'k-', ttb, xbb, 'k--', ttc, xcc, 'k-.', ...
  ttd, xdd, 'b-', tte, xee, 'b--', ttf, xff, 'b-.');
xlabel ('t'); ylabel('x'); axis square; title('t-x');
subplot(132),
plot(tta, yaa, 'k-', ttb, ybb, 'k--', ttc, ycc, 'k-.', ...
  ttd, ydd, 'b-', tte, yee, 'b--', ttf, yff, 'b-.')
xlabel ('t'); ylabel('y'); axis square; title('t-y');
subplot(133),
plot(xaa, yaa, 'k-', xbb, ybb, 'k--', xcc, ycc, 'k-.', ...
  xdd, ydd, 'b-', xee, yee, 'b--', xff, yff, 'b-.'); hold on;
scatter(X, Y, 20, 'r', 'filled');
xlabel ('x'); ylabel('y'); axis square; grid; title('x-y');
axis([0 9 -2 6]);
d = 0.5;
text(x0, y0+d, 'P_0');
text(x1, y1-d, 'P_1');
text(x2, y2-d, 'P_2');
text(x3, y3+d, 'P_3');
hold off;
```

注解

…：将当前命令或函数调用继续到下一行。

simplify：通过解决所有交集和嵌套来简化方程。

3.5 结向量值的证明

现在，是时候为选择我们迄今为止一直使用的[0,1,2,3,…]的 KV 值提供理由了。让我们首先选择其他一组值并观察我们得到的结果。因此，让我们假设对于均匀二次 B 样条，将 KV 更改为 $\boldsymbol{T}$=[0,5,10,15,20,25,30]。我们可以任意选择任何一组值，唯一的约束是值之间的间距应该是一致的，这也是对均匀 **B 样条**的要求。因此，在这种情况下，我们选择了一个间距为早期值 5 倍的 KV。如果我们按照与 3.4 节相同的步骤进行并计算 BF，会得到如下所示的结果：

$$\begin{cases} B_{0,3}=\begin{cases}(1/50)t^2 & (0\leqslant t<1)A\\ -t^2/25+3t/5-(3/2) & (1\leqslant t<2)B\\ (1/50)(t-15)^2 & (2\leqslant t<3)C\end{cases}\\ B_{1,3}=\begin{cases}(1/50)(t-5)^2 & (1\leqslant t<2)B\\ -t^2/25+t-(11/2) & (2\leqslant t<3)C\\ (1/50)(t-20)^2 & (3\leqslant t<4)D\end{cases}\\ B_{2,3}=\begin{cases}(1/50)(t-10)^2 & (2\leqslant t<3)C\\ -t^2/25+7t/5-(23/2) & (3\leqslant t<4)D\\ (1/50)(t-25)^2 & (4\leqslant t<5)E\end{cases}\\ B_{3,3}=\begin{cases}(1/50)(t-15)^2 & (3\leqslant t<4)D\\ -t^2/25+9t/5-(39/2) & (4\leqslant t<5)E\\ (1/50)(t-30)^2 & (5\leqslant t<6)F\end{cases}\end{cases} \tag{3.18}$$

与式(3.14)比较可知，t 已被 $t/5$ 替换，即所有 t 值都已缩放 5 倍。所以现在要获得相同的 B 值需要 t 值比之前大 5 倍。这反映在图 3.7 中的 BF 图中，与图 3.5 中的 BF 图进行比较，它显示的 t 轴要扩大 5 倍。

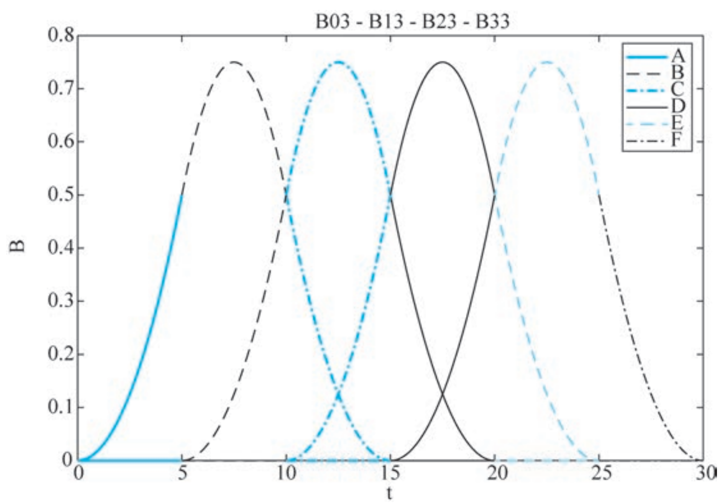

图 3.7 BF 随 KV 的变化而变化

将新的 KV 插入例 3.2 后，我们可以观察到对参数曲线的影响。可以看到下面显示的 $x(t)$和 $y(t)$的值受到类似于 BF 的影响，即这些值已按 5 倍进行了缩放。

$$x(t)=\begin{cases}t^2/50 & (0\leqslant t<5)\\ t^2/25-t/5+1/2 & (5\leqslant t<10)\\ -t^2/50+t-11/2 & (10\leqslant t<15)\\ 2t/5-1 & (15\leqslant t<20)\\ -t^2/5+42t/5-81 & (20\leqslant t<25)\\ (4/25)(t-30)^2 & (25\leqslant t<30)\end{cases}$$

$$y(t)=\begin{cases} t^2/25 & (0\leqslant t<5)\\ -3t^2/50+t-5/2 & (5\leqslant t<10)\\ t^2/10-11t/5+27/2 & (10\leqslant t<15)\\ -t^2/5+34t/5-54 & (15\leqslant t<20)\\ 7t^2/50-34t/5+82 & (20\leqslant t<25)\\ -(1/50)(t-30)^2 & (25\leqslant t<30)\end{cases}$$

在绘制实际曲线时，可以看到 x-t 和 y-t 图受到类似影响，即 t 值已更改 5 倍。请参见图 3.8，不同的线型表示分段间隔。然而，由于 x 和 y 值保持不变，x-y 图与以前完全相同，因为它不受 t 值的任何变化的影响，只要它以相同的量均匀地影响 x 和 y 值。例如，对于图 3.6，有 $t=2$，$x=2.5$，$y=1.5$，而对于图 3.8，有 $t=10$，$x=2.5$，$y=1.5$，这意味着 x 与 y 保持不变，即不受影响。这对所有点都是正确的，因此曲线的 x-y 图与以前相同。显然，对于任何改变 t 的比例因子都是如此。

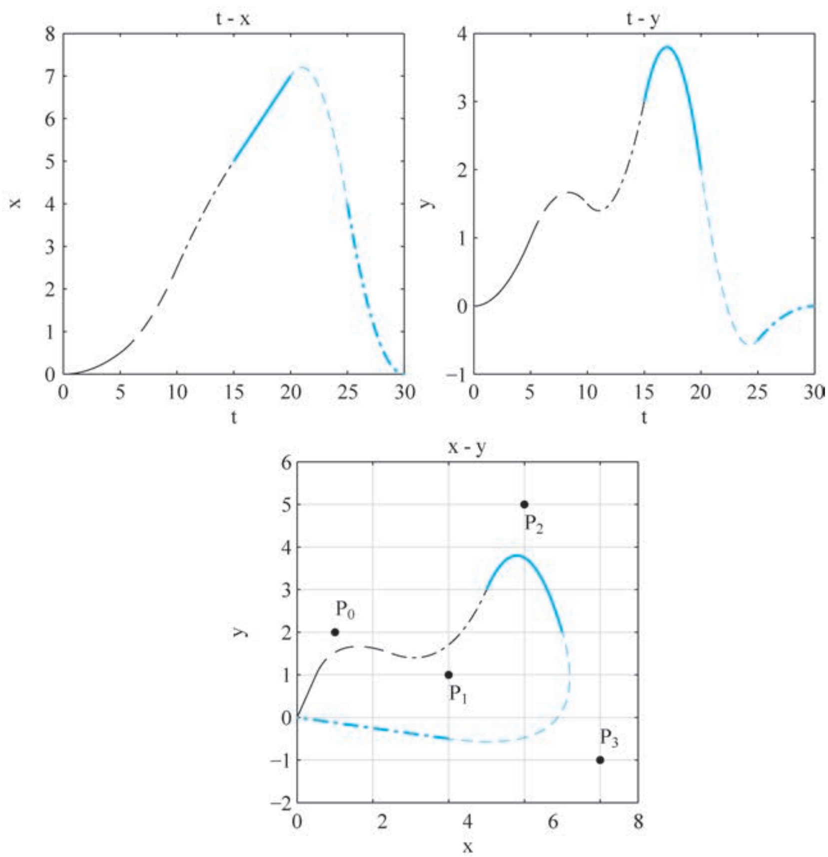

图 3.8　曲线方程随 KV 变化的变化

鼓励读者使用不同的 KV 值生成图表并验证结果，可以通过使用 MATLAB Code 3.2 并简单地更改程序第二行中的 KV 来轻松地做到这一点。

这使我们得出结论，空间域中的实际曲线与 KV 值无关，因为它不依赖于 t 值的比例因子。因此，习惯上选择最小的 t 值，即 0，1，2，…以降低方程的复杂性，但实际上，我们可以

为 KV 选择任何值并得到相同的结果。

3.6　二次开放均匀 B 样条

在开放均匀样条曲线中，KV 是均匀的，除了重复 d 次的末端，其中$(d-1)$是曲线的次数。重复值称为多重性。多重性意味着 Cox de Boor 项的分母在许多情况下都为零。因此，这里需要做一个重要的假设：除以零被视为零。

考虑一个 $d=3$ 和 $n=3$ 的二次开放均匀 B 样条。让 KV 选择为 $\boldsymbol{T}=\{1,1,1,2,3,3,3\}$。如前所述，进行分段分析，获得的结果列于表 3.3 中。

表 3.3　二次开放均匀 B 样条的 BF 计算

段	t	$B_{k,1}$	$B_{k,2}$	$B_{k,3}$
A	$0\leqslant t<1$	$B_{0,1}=1$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
B	$1\leqslant t<2$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=1$	$B_{1,2}=0$	$B_{1,3}=0$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
C	$2\leqslant t<3$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=(t-2)^2$
		$B_{1,1}=0$	$B_{1,2}=2-t$	$B_{1,3}=-(3/2)t^2+5t-(7/2)$
		$B_{2,1}=1$	$B_{2,2}=t-1$	$B_{2,3}=(1/2)(t-1)^2$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
D	$3\leqslant t<4$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=(1/2)(t-3)^2$
		$B_{2,1}=0$	$B_{2,2}=3-t$	$B_{2,3}=-(3/2)t^2+7t-(15/2)$
		$B_{3,1}=1$	$B_{3,2}=t-2$	$B_{3,3}=(t-2)^2$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=0$		
E	$4\leqslant t<5$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=1$	$B_{4,2}=0$	
		$B_{5,1}=0$		
F	$5\leqslant t<6$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$

续表

段	t	$B_{k,1}$	$B_{k,2}$	$B_{k,3}$
F	$5 \leqslant t < 6$	$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$
		$B_{4,1}=0$	$B_{4,2}=0$	
		$B_{5,1}=1$		

将上述值代入式(3.13)，得到：

$$\begin{cases} B_{0,3}=(t-2)^2 & (2 \leqslant t < 3)C \\ B_{1,3}=\begin{cases} -(3/2)t^2+5t-(7/2) & (2 \leqslant t < 3)C \\ (1/2)(t-3)^2 & (3 \leqslant t < 4)D \end{cases} \\ B_{2,3}=\begin{cases} (1/2)(t-1)^2 & (2 \leqslant t < 3)C \\ -(3/2)t^2+7t-(15/2) & (3 \leqslant t < 4)D \end{cases} \\ B_{3,3}=(t-2)^2 & (3 \leqslant t < 4)D \end{cases} \tag{3.19}$$

式(3.19)表示具有 4 个 CP 的二次开放均匀 B 样条的 BF。BF 的图如图 3.9 所示。仅存在与段 $C(2\leqslant t<3)$和 $D(3\leqslant t<4)$相关的部分。

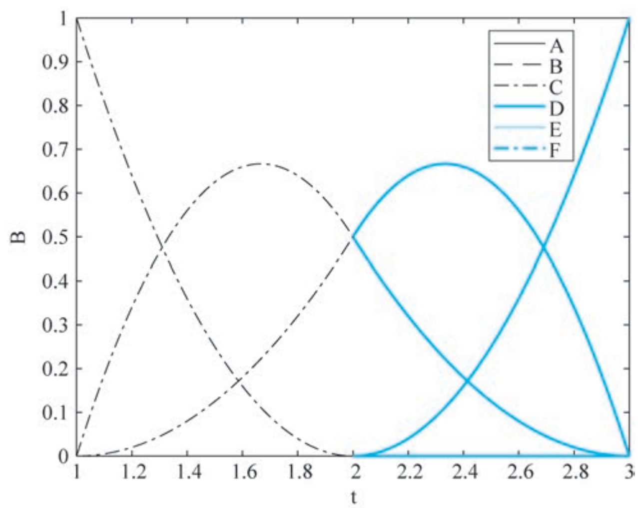

图 3.9　二次开放均匀 B 样条的 BF

曲线方程是通过将 BF 值代入式(3.15)得到的：

$$P(t)=\begin{cases} 0 & (0 \leqslant t < 1) \\ 0 & (1 \leqslant t < 2) \\ P_0(t-2)^2+P_1\left(-\dfrac{3}{2}t^2+5t-\dfrac{7}{2}\right)+P_2\left(\dfrac{1}{2}\right)(t-1)^2 & (2 \leqslant t < 3) \\ P_1\left(\dfrac{1}{2}\right)(t-3)^2+P_2\left(-\dfrac{3}{2}t^2+7t-\dfrac{15}{2}\right)+P_3(t-2)^2 & (3 \leqslant t < 4) \\ 0 & (4 \leqslant t < 5) \\ 0 & (5 \leqslant t < 6) \end{cases} \tag{3.20}$$

图 3.10 中的曲线图显示了多重性对样条曲线的影响：它迫使曲线实际通过第一个和最后一个 CP。这创建了一条只有两条线段 C 和 D 的样条曲线，而其他线段 A、B、E 和 F 不存在。因此，我们可以得出结论，若 KV 具有重复值，则近似样条可以表现得像插值样条或混合样条那样。

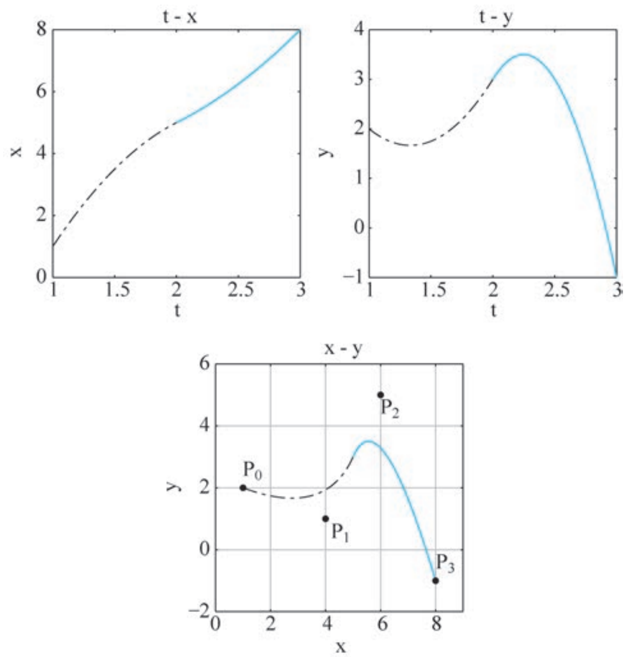

图 3.10　开放均匀二次 B 样条曲线

例 3.3　求具有 CP，即 $P_0(1,2)$、$P_1(4,1)$、$P_2(6,5)$和 $P_3(8,-1)$的开放均匀二次 B 样条方程。还要编写一个程序来绘制 BF 和实际曲线。

解：

根据式(3.20)，代入给定 CP 的值得到：

$$x(t)=\begin{cases}0 & (0\leqslant t<1)\\ 0 & (1\leqslant t<2)\\ -2t^2+10t-7 & (2\leqslant t<3)\\ t^2-2t+5 & (3\leqslant t<4)\\ 0 & (4\leqslant t<5)\\ 0 & (5\leqslant t<6)\end{cases}$$

$$y(t)=\begin{cases}0 & (0\leqslant t<1)\\ 0 & (1\leqslant t<2)\\ 3t^2-8t+7 & (2\leqslant t<3)\\ -8t^2+36t-37 & (3\leqslant t<4)\\ 0 & (4\leqslant t<5)\\ 0 & (5\leqslant t<6)\end{cases}$$

MATLAB Code 3.3

该代码与 MATLAB Code 3.2 几乎完全相同，除了在计算每个 BF 之前进行额外检查以避免被零除的情况。如果存在被零除条件，则表达式将替换为 0；否则，应按正常流程计算。说明如下。

```
D = (t1 - t0); if D = = 0, S1 = 0; else S1 = (t - t0)/D; end;
D = (t2 - t1); if D = = 0, S2 = 0; else S2 = (t - t0)/D; end;
B02 = S1 * B01 + S2 * B11; B02A = B02;
```

3.7 二次非均匀 B 样条

在非均匀样条中，KV 不均匀，即结点元素之间的间距不同。这使得 BF 不对称，并且它们倾向于在间距较小的地方聚集在一起，这具有将曲线拉向相应 CP 的效果。图 3.11 显示了二次非均匀 B 样条的 BF。不对称的程度取决于 KV 中的间距。

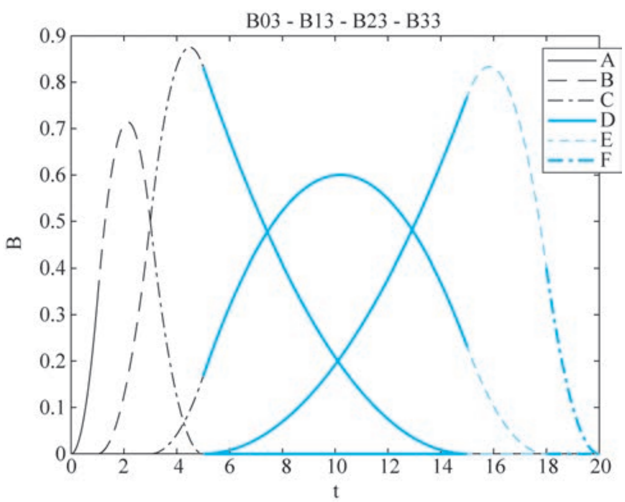

图 3.11 二次非均匀 B 样条的 BF

例 3.4 求具有 CP，即 $P_0(1,2)$、$P_1(4,1)$、$P_2(6,5)$和 $P_3(8,-1)$的二次非均匀 B 样条方程，假设 KV 为 $\boldsymbol{T}=[0,1,3,5,15,18,20]$。

解：

使用分段分析：

$$x(t)=\begin{cases} t^2/3 & (0\leqslant t<1) \\ 5t^2/24+t/4-1/8 & (1\leqslant t<2) \\ -7t^2/24+13t/4-37/8 & (2\leqslant t<3) \\ -t^2/780+9t/26+137/52 & (3\leqslant t<4) \\ -38t^2/65+232t/13-1672/13 & (4\leqslant t<5) \\ 4(t-20)^2/5 & (5\leqslant t<6) \end{cases}$$

$$y(t)=\begin{cases}2t^2/3 & (0\leqslant t<1)\\ -11t^2/24+9t/4-9/8 & (1\leqslant t<2)\\ 7t^2/24-9t/4+45/8 & (2\leqslant t<3)\\ -t^2/390+9t/13+95/26 & (3\leqslant t<4)\\ 43t^2/195-98t/13+830/13 & (4\leqslant t<5)\\ -(t-20)^2/10 & (5\leqslant t<6)\end{cases}$$

MATLAB Code 3.4

与 MATLAB Code 3.2 相同，但 KV 有所变化。

3.8 三次均匀 B 样条

三次均匀 B 样条的处理方式与二次均匀 B 样条的处理方式大致相同，但由 **Cox de Boor 算法**生成的四阶 BF 项产生了额外的复杂性。

为了生成三次均匀 B 样条，我们需要从 $d=4$ 和 $n=4$ 开始：

曲线的阶数：$d-1=3$。

CP 数量：$n+1=5$。

曲线段数：$d+n=8$。

KV 中的元素数：$d+n+1=9$。

设曲线段为 A、B、C、D、E、F、G 和 H，CP 为 P_0、P_1、P_2、P_3 和 P_4（见图 3.12）。

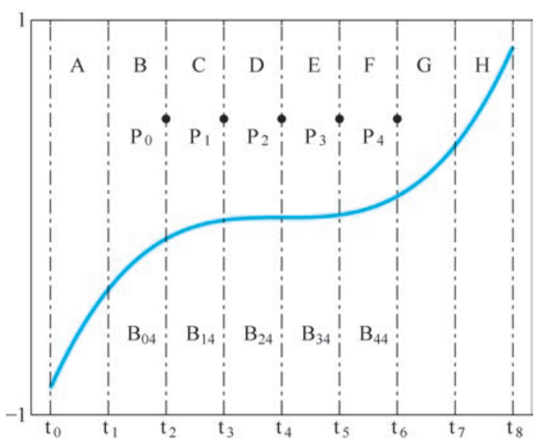

图 3.12 具有 5 个 CP 的三次均匀 B 样条

对于 $k=\{0,1,2,3,4,5,6,7,8\}$，令 KV 为 $\boldsymbol{T}=\{t_k\}$。在这种情况下，$\boldsymbol{T}=[t_0,t_1,t_2,t_3,t_4,t_5,t_6,t_7,t_8]$。

设 BF 为 $B_{0,4}$、$B_{1,4}$、$B_{2,4}$、$B_{3,4}$ 和 $B_{4,4}$。

曲线方程由下式给出：

$$P(t)=P_0B_{0,4}+P_1B_{1,4}+P_2B_{2,4}+P_3B_{3,4}+P_4B_{4,4} \tag{3.21}$$

如前所述，我们假设 KV 为 $\boldsymbol{T}=[0,1,2,3,4,5,6,7,8]$。根据第一个 Cox de Boor 算法的条件，一阶项 $B_{0,1}$，$B_{1,1}$，$B_{2,1}$，$B_{3,1}$，$B_{4,1}$，$B_{5,1}$，$B_{6,1}$，$B_{7,1}$ 和 $B_{8,1}$ 将为 0 或 1。二阶项计

算如下：

$$\begin{cases}B_{0,2}=(t-0)B_{0,1}+(2-t)B_{1,1}\\B_{1,2}=(t-1)B_{1,1}+(3-t)B_{2,1}\\B_{2,2}=(t-2)B_{2,1}+(4-t)B_{3,1}\\B_{3,2}=(t-3)B_{3,1}+(5-t)B_{4,1}\\B_{4,2}=(t-4)B_{4,1}+(6-t)B_{5,1}\\B_{5,2}=(t-5)B_{5,1}+(7-t)B_{6,1}\\B_{6,2}=(t-6)B_{6,1}+(8-t)B_{7,1}\\B_{7,2}=(t-7)B_{7,1}+(9-t)B_{8,1}\end{cases}\tag{3.22}$$

三阶项由二阶项计算得出：

$$\begin{cases}B_{0,3}=(1/2)(t-0)B_{0,2}+(1/2)(3-t)B_{1,2}\\B_{1,3}=(1/2)(t-1)B_{1,2}+(1/2)(4-t)B_{2,2}\\B_{2,3}=(1/2)(t-2)B_{2,2}+(1/2)(5-t)B_{3,2}\\B_{3,3}=(1/2)(t-3)B_{3,2}+(1/2)(6-t)B_{4,2}\\B_{4,3}=(1/2)(t-4)B_{4,2}+(1/2)(7-t)B_{5,2}\\B_{5,3}=(1/2)(t-5)B_{5,2}+(1/2)(8-t)B_{6,2}\\B_{6,3}=(1/2)(t-6)B_{6,2}+(1/2)(9-t)B_{7,2}\end{cases}\tag{3.23}$$

四阶项由三阶项计算得出：

$$\begin{cases}B_{0,4}=(1/3)(t-0)B_{0,3}+(1/3)(4-t)B_{1,3}\\B_{1,4}=(1/3)(t-1)B_{1,3}+(1/3)(5-t)B_{2,3}\\B_{2,4}=(1/3)(t-2)B_{2,3}+(1/3)(6-t)B_{3,3}\\B_{3,4}=(1/3)(t-3)B_{3,3}+(1/3)(7-t)B_{4,3}\\B_{4,4}=(1/3)(t-4)B_{4,3}+(1/3)(8-t)B_{5,3}\end{cases}\tag{3.24}$$

由于有 8 个段，每个 BF 包含 8 个子分量：

$$\begin{cases}B_{0,4}=\{B_{0,4A},B_{0,4B},B_{0,4C},B_{0,4D},B_{0,4E},B_{0,4F},B_{0,4G},B_{0,4H}\}\\B_{1,4}=\{B_{1,4A},B_{1,4B},B_{1,4C},B_{1,4D},B_{1,4E},B_{1,4F},B_{1,4G},B_{1,4H}\}\\B_{2,4}=\{B_{2,4A},B_{2,4B},B_{2,4C},B_{2,4D},B_{2,4E},B_{2,4F},B_{2,4G},B_{2,4G}\}\\B_{3,4}=\{B_{3,4A},B_{3,4B},B_{3,4C},B_{3,4D},B_{3,4E},B_{3,4F},B_{3,4G},B_{3,4H}\}\\B_{4,4}=\{B_{4,4A},B_{4,4B},B_{4,4C},B_{4,4D},B_{4,4E},B_{4,4F},B_{4,4G},B_{4,4H}\}\end{cases}\tag{3.25}$$

表 3.4 总结了对 BF 值的计算。

表 3.4　三次均匀 B 样条的 BF 计算

段	t	$B_{k,1}$	$B_{k,2}$	$B_{k,3}$	$B_{k,4}$
A	$0\leqslant t<1$	$B_{0,1}=1$	$B_{0,2}=t$	$B_{0,3}=(1/2)t^2$	$B_{0,4}=(1/6)t^3$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$	
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$	

续表

段	t	$B_{k,1}$	$B_{k,2}$	$B_{k,3}$	$B_{k,4}$
A	$0\leqslant t<1$	$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$	
		$B_{4,1}=0$	$B_{4,2}=0$		
		$B_{5,1}=0$			
		$B_{6,1}=0$			
		$B_{7,1}=0$			
B	$1\leqslant t<2$	$B_{0,1}=0$	$B_{0,2}=2-t$	$B_{0,3}=-t^2+3t-(3/2)$	$B_{0,4}=-(1/3)t^3+2t^2-2t+(2/3)$
		$B_{1,1}=1$	$B_{1,2}=t-1$	$B_{1,3}=(1/2)(t-1)^2$	$B_{1,4}=(1/6)(t-1)^3$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$	
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$	
		$B_{4,1}=0$	$B_{4,2}=0$		
		$B_{5,1}=0$			
		$B_{6,1}=0$			
		$B_{7,1}=0$			
C	$2\leqslant t<3$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=(1/2)(t-3)^2$	$B_{0,4}=-(1/2)t^3-4t^2+10t-(22/3)$
		$B_{1,1}=0$	$B_{1,2}=3-t$	$B_{1,3}=-t^2+5t-(11/2)$	$B_{1,4}=-(1/2)t^3+(7/2)t^2-(15/2)t+(31/6)$
		$B_{2,1}=1$	$B_{2,2}=t-2$	$B_{2,3}=(1/2)(t-2)^2$	$B_{2,4}=(1/6)(t-2)^3$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$	
		$B_{4,1}=0$	$B_{4,2}=0$		
		$B_{5,1}=0$			
		$B_{6,1}=0$			
		$B_{7,1}=0$			
D	$3\leqslant t<4$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$	$B_{0,4}=-(1/6)(t-4)^3$
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=(1/2)(t-4)^2$	$B_{1,4}=(1/2)t^3-(11/2)t^2+(39/2)t-(131/6)$
		$B_{2,1}=0$	$B_{2,2}=4-t$	$B_{2,3}=-t^2+7t-(23/2)$	$B_{2,4}=-(1/2)t^3+(5/2)t^2-16t+(50/3)$
		$B_{3,1}=1$	$B_{3,2}=t-3$	$B_{3,3}=(1/2)(t-3)^2$	$B_{3,4}=(1/6)(t-3)^3$
		$B_{4,1}=0$	$B_{4,2}=0$		
		$B_{5,1}=0$			
		$B_{6,1}=0$			
		$B_{7,1}=0$			

续表

段	t	$B_{k,1}$	$B_{k,2}$	$B_{k,3}$	$B_{k,4}$
E	$4\leqslant t<5$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$	
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$	$B_{1,4}=-(1/6)(t-5)^3$
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=(1/2)(t-5)^2$	$B_{2,4}=(1/2)t^3-(7/2)t^2+32t-(142/3)$
		$B_{3,1}=0$	$B_{3,2}=5-t$	$B_{3,3}=-t^2+9t-(39/2)$	$B_{3,4}=-(1/2)t^3+(13/2)t^2-(55/2)t+(229/6)$
		$B_{4,1}=1$	$B_{4,2}=t-4$	$B_{4,3}=(1/2)(t-4)^2$	$B_{4,4}=(1/6)(t-4)^3$
		$B_{5,1}=0$			
		$B_{6,1}=0$			
		$B_{7,1}=0$			
F	$5\leqslant t<6$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$	
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$	
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$	$B_{2,4}=-(1/6)(t-6)^3$
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=(1/2)(t-6)^2$	$B_{3,4}=(1/2)t^3-(17/2)t^2+(95/2)t-(521/6)$
		$B_{4,1}=0$	$B_{4,2}=6-t$	$B_{4,3}=-t^2+11t-(59/2)$	$B_{4,4}=-(1/2)t^3+8t^2-42t+(218/3)$
		$B_{5,1}=1$	$B_{5,2}=t-5$	$B_{5,3}=(1/2)(t-5)^2$	
		$B_{6,1}=0$			
		$B_{7,1}=0$			
G	$6\leqslant t<7$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$	
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$	
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$	
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$	$B_{3,4}=-(1/6)(t-7)^3$
		$B_{4,1}=0$	$B_{4,2}=0$	$B_{4,3}=(1/2)(t-7)^2$	$B_{4,4}=(1/2)t^3-10t^2+66t-(430/3)$
		$B_{5,1}=0$	$B_{5,2}=7-t$	$B_{5,3}=-t^2+13t-(83/2)$	
		$B_{6,1}=1$	$B_{6,2}=t-6$	$B_{6,3}=(1/2)(t-6)^2$	
		$B_{7,1}=0$			
H	$7\leqslant t<8$	$B_{0,1}=0$	$B_{0,2}=0$	$B_{0,3}=0$	
		$B_{1,1}=0$	$B_{1,2}=0$	$B_{1,3}=0$	
		$B_{2,1}=0$	$B_{2,2}=0$	$B_{2,3}=0$	
		$B_{3,1}=0$	$B_{3,2}=0$	$B_{3,3}=0$	
		$B_{4,1}=0$	$B_{4,2}=0$	$B_{4,3}=0$	$B_{4,4}=-(1/6)(t-8)^3$
		$B_{5,1}=0$	$B_{5,2}=0$	$B_{5,3}=(1/2)(t-8)^2$	
		$B_{6,1}=0$	$B_{6,2}=8-t$	$B_{6,3}=-(1/2)t^2+7t-24$	
		$B_{7,1}=1$	$B_{7,2}=t-7$		

将上述值代入式(3.25)，可以得到：

$$
\begin{cases}
B_{0,4}=\begin{cases}(1/6)t^3 & (0\leqslant t<1)A\\ -(1/3)t^3+2t^2-2t+(2/3) & (1\leqslant t<2)B\\ -(1/2)t^3-4t^2+10t-(22/3) & (2\leqslant t<3)C\\ -(1/6)(t-4)^3 & (3\leqslant t<4)D\end{cases}\\
B_{1,4}=\begin{cases}(1/6)(t-1)^3 & (1\leqslant t<2)B\\ -(1/2)t^3+(7/2)t^2-(15/2)t+(31/6) & (2\leqslant t<3)C\\ (1/2)t^3-(11/2)t^2+(39/2)t-(131/6) & (3\leqslant t<4)D\\ -(1/6)(t-5)^3 & (4\leqslant t<5)E\end{cases}\\
B_{2,4}=\begin{cases}(1/6)(t-2)^3 & (2\leqslant t<3)C\\ -(1/2)t^3+(5/2)t^2-16t+(50/3) & (3\leqslant t<4)D\\ (1/2)t^3-(7/2)t^2+32t-(142/3) & (4\leqslant t<5)E\\ -(1/6)(t-6)^3 & (5\leqslant t<6)F\end{cases}\\
B_{3,4}=\begin{cases}(1/6)(t-3)^3 & (3\leqslant t<4)D\\ -(1/2)t^3+(13/2)t^2-(55/2)t+(229/6) & (4\leqslant t<5)E\\ (1/2)t^3-(17/2)t^2+(95/2)t-(521/6) & (5\leqslant t<6)F\\ -(1/6)(t-7)^3 & (6\leqslant t<7)G\end{cases}\\
B_{4,4}=\begin{cases}(1/6)(t-4)^3 & (4\leqslant t<5)E\\ -(1/2)t^3+8t^2-42t+(218/3) & (5\leqslant t<6)F\\ (1/2)t^3-10t^2+66t-(430/3) & (6\leqslant t<7)G\\ -(1/6)(t-8)^3 & (7\leqslant t<8)H\end{cases}
\end{cases}
\tag{3.26}
$$

式(3.26)表示具有5个CP和8个段的三次均匀B样条的BF。BF的图如图3.13所示。每个BF具有相同的形状,但相对于前一个向右移动1。因此,每个BF可以通过将t替换为$(t-1)$从前一个BF获得。如式(3.26)所示,每个BF有8个细分,其中4个是非零的。$B_{0,4}$的第一条曲线对于分段$A(0\leqslant t<1)$、$B(1\leqslant t<2)$、$C(2\leqslant t<3)$和$D(3\leqslant t<4)$具有非零部分,$B_{1,4}$的第二条曲线对于分段$B(1\leqslant t<2)$、$C(2\leqslant t<3)$、$D(3\leqslant t<4)$和$E(4\leqslant t<5)$具有非零部分,$B_{2,4}$的第三条曲线对于分段$C(2\leqslant t<3)$、$D(3\leqslant t<4)$、$E(4\leqslant t<5)$和$F(5\leqslant t<6)$具有非零部分,$B_{3,4}$的第四条曲线对于分段$D(3\leqslant t<4)$、$E(4\leqslant t<5)$、$F(5\leqslant t<6)$和$G(6\leqslant t<7)$具有非零部分,$B_{4,4}$的第五条曲线对于分段$E(4\leqslant t<5)$、$F(5\leqslant t<6)$、$G(6\leqslant t<7)$和$H(7\leqslant t<8)$具有非零部分。由于BF与CP相关联,因此这提供了三次样条的局部控制属性的指示。第一个CP对前4个段A、B、C和D有影响,第二个CP对B、C、D和E有影响,以此类推。这意味着如果第一个CP发生更改,它将仅影响前4个段,而样条的其余部分将保持不变。

样条方程是其八段方程的集合:

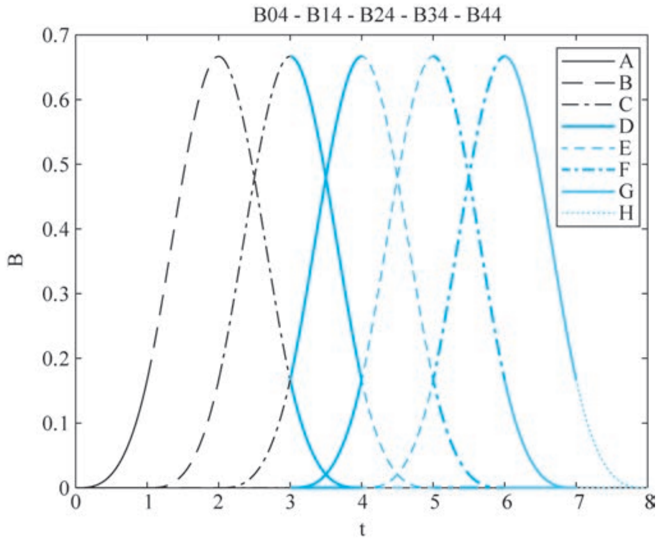

图 3.13　具有 5 个 CP 的三次均匀 B 样条的 BF

$$P(t)=\begin{cases} P_A & (0 \leqslant t < 1) \\ P_B & (1 \leqslant t < 2) \\ P_C & (2 \leqslant t < 3) \\ P_D & (3 \leqslant t < 4) \\ P_E & (4 \leqslant t < 5) \\ P_F & (5 \leqslant t < 6) \\ P_G & (6 \leqslant t < 7) \\ P_H & (7 \leqslant t < 8) \end{cases} \tag{3.27}$$

其中，

$$\begin{cases} P_A = P_0 B_{0,4A} + P_1 B_{1,4A} + P_2 B_{2,4A} + P_3 B_{3,4A} + P_4 B_{4,4A} \\ P_B = P_0 B_{0,4B} + P_1 B_{1,4B} + P_2 B_{2,4B} + P_3 B_{3,4B} + P_4 B_{4,4B} \\ P_C = P_0 B_{0,4C} + P_1 B_{1,4C} + P_2 B_{2,4C} + P_3 B_{3,4C} + P_4 B_{4,4C} \\ P_D = P_0 B_{0,4D} + P_1 B_{1,4D} + P_2 B_{2,4D} + P_3 B_{3,4D} + P_4 B_{4,4D} \\ P_E = P_0 B_{0,4E} + P_1 B_{1,4E} + P_2 B_{2,4E} + P_3 B_{3,4E} + P_4 B_{4,4E} \\ P_F = P_0 B_{0,4F} + P_1 B_{1,4F} + P_2 B_{2,4F} + P_3 B_{3,4F} + P_4 B_{4,4F} \\ P_G = P_0 B_{0,4G} + P_1 B_{1,4G} + P_2 B_{2,4G} + P_3 B_{3,4G} + P_4 B_{4,4G} \\ P_H = P_0 B_{0,4H} + P_1 B_{1,4H} + P_2 B_{2,4H} + P_3 B_{3,4H} + P_4 B_{4,4H} \end{cases} \tag{3.28}$$

将表 3.4 中的 BF 值代入式(3.28)，可以得到：

$$P(t) = \begin{cases} P_0\left(\frac{1}{6}\right)t^3 \\ P_0\left(-\frac{t^3}{3}+2t^2-2t+\frac{2}{3}\right)+P_1\left(\frac{1}{6}\right)(t-1)^3 \\ P_0\left(-\frac{t^3}{2}-4t^2+10t-\frac{22}{3}\right)+P_1\left(-\frac{t^3}{2}+\frac{7t^2}{2}-\frac{15t}{2}+\frac{31}{6}\right)+P_2\left(\frac{1}{6}\right)(t-2)^3 \\ P_0\left(\frac{1}{6}\right)(4-t)^3+P_1\left(\frac{t^3}{2}-\frac{11t^2}{2}+\frac{39t}{2}-\frac{131}{6}\right)+P_2\left(-\frac{t^3}{2}+\frac{5t^2}{2}-16t+\frac{50}{3}\right)+P_3\left(\frac{1}{6}\right)(t-3)^3 \\ P_1\left(\frac{1}{6}\right)(5-t)^3+P_2\left(\frac{t^3}{2}-\frac{7t^2}{2}+32t-\frac{142}{3}\right)+P_3\left(-\frac{t^3}{2}+\frac{13t^2}{2}-\frac{55t}{2}+\frac{229}{6}\right)+P_4\left(\frac{1}{6}\right)(t-4)^3 \\ P_2\left(\frac{1}{6}\right)(6-t)^3+P_3\left(\frac{t^3}{2}-\frac{17t^2}{2}+\frac{95t}{2}-\frac{521}{6}\right)+P_4\left(-\frac{t^3}{2}+8t^2-42t+\frac{218}{3}\right) \\ P_3\left(\frac{1}{6}\right)(7-t)^3+P_4\left(\frac{t^3}{2}-10t^2+66t-\frac{430}{3}\right) \\ P_4\left(\frac{1}{6}\right)(8-t)^3 \end{cases} \tag{3.29}$$

式(3.29)表示具有 5 个 CP 的三次均匀 B 样条曲线方程。方程的 8 个子分量代表 8 个段的部分。

例 3.5 找到具有 CP，即(−1,0)、(0,1)、(1,0)、(0,−1)和(−0.5,−0.5)的均匀三次 B 样条的方程。还要编写一个程序来绘制其 BF 和实际曲线。

解：

将得到的给定 CP 值代入式(3.29)，得到(见图 3.14)。

$$x(t) = \begin{cases} t^3/6 & (0 \leqslant t < 1) \\ t^3/3 - 2t^2 + 2t - 2/3 & (1 \leqslant t < 2) \\ 2t^3/3 + 3t^2 - 32/3t + 6 & (2 \leqslant t < 3) \\ -t^3/3 + t^2/2 - 8t + 6 & (3 \leqslant t < 4) \\ \frac{7}{12}t^3 - 5t^2 + \frac{81}{2}t - 125 & (4 \leqslant t < 5) \\ \frac{1}{12}t^3 - 2t^2 + 9t - 47 & (5 \leqslant t < 6) \\ -t^3/4 + 5t^2 - 33t + 215/3 & (6 \leqslant t < 7) \\ (t-8)^3/12 & (7 \leqslant t < 8) \end{cases}$$

$$y(t) = \begin{cases} 0 & (0 \leqslant t < 1) \\ (t-1)^3/6 & (1 \leqslant t < 2) \\ -t^3/2 + 7t^2/2 - 15t/2 + 31/6 & (2 \leqslant t < 3) \\ t^3/3 - 4t^2 + 15t - 52/3 & (3 \leqslant t < 4) \\ t^3/4 - 3t^2 + 11t - 12 & (4 \leqslant t < 5) \\ -t^3/4 + 9t^2/2 - 53t/2 + 101/2 & (5 \leqslant t < 6) \\ -t^3/12 + 3t^2/2 - 17t/2 + 29/2 & (6 \leqslant t < 7) \\ (t-8)^3/12 & (7 \leqslant t < 8) \end{cases}$$

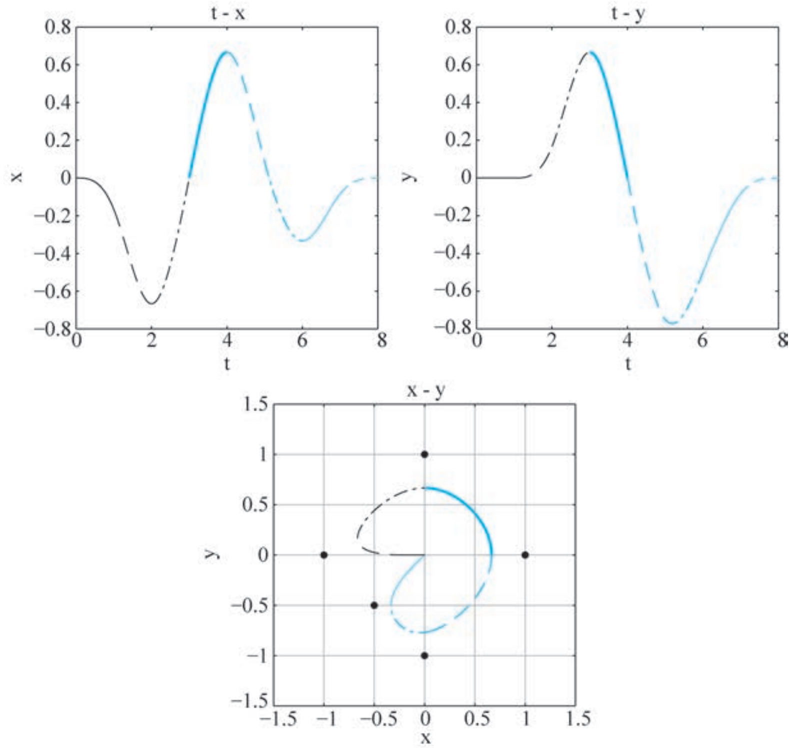

图 3.14　例 3.5 的绘图

MATLAB Code 3.5

```
clear all; format compact; clc;
x0 = -1; x1 = 0; x2 = 1; x3 = 0; x4 = -0.5;
y0 = 0; y1 = 1; y2 = 0; y3 = -1; y4 = -0.5;
t0 = 0; t1 = 1; t2 = 2; t3 = 3; t4 = 4; t5 = 5; t6 = 6; t7 = 7; t8 = 8;
T = [t0, t1, t2, t3, t4, t5, t6, t7, t8];
syms t P0 P1 P2 P3 P4;
syms B01 B11 B21 B31 B41 B51 B61 B71;
B02 = ((t - t0)/(t1 - t0)) * B01 + ((t2 - t)/(t2 - t1)) * B11;
B12 = ((t - t1)/(t2 - t1)) * B11 + ((t3 - t)/(t3 - t2)) * B21;
B22 = ((t - t2)/(t3 - t2)) * B21 + ((t4 - t)/(t4 - t3)) * B31;
B32 = ((t - t3)/(t4 - t3)) * B31 + ((t5 - t)/(t5 - t4)) * B41;
B42 = ((t - t4)/(t5 - t4)) * B41 + ((t6 - t)/(t6 - t5)) * B51;
B52 = ((t - t5)/(t6 - t5)) * B51 + ((t7 - t)/(t7 - t6)) * B61;
B62 = ((t - t6)/(t7 - t6)) * B61 + ((t8 - t)/(t8 - t7)) * B71;
B72 = ((t - t7)/(t8 - t7)) * B71 + 0;
B03 = ((t - t0)/(t2 - t0)) * B02 + ((t3 - t)/(t3 - t1)) * B12;
B13 = ((t - t1)/(t3 - t1)) * B12 + ((t4 - t)/(t4 - t2)) * B22;
B23 = ((t - t2)/(t4 - t2)) * B22 + ((t5 - t)/(t5 - t3)) * B32;
B33 = ((t - t3)/(t5 - t3)) * B32 + ((t6 - t)/(t6 - t4)) * B42;
B43 = ((t - t4)/(t6 - t4)) * B42 + ((t7 - t)/(t7 - t5)) * B52;
B53 = ((t - t5)/(t7 - t5)) * B52 + ((t8 - t)/(t8 - t6)) * B62;
B63 = ((t - t6)/(t8 - t6)) * B62 + 0;
B04 = ((t - t0)/(t3 - t0)) * B03 + ((t4 - t)/(t4 - t1)) * B13;
B14 = ((t - t1)/(t4 - t1)) * B13 + ((t5 - t)/(t5 - t2)) * B23;
B24 = ((t - t2)/(t5 - t2)) * B23 + ((t6 - t)/(t6 - t3)) * B33;
```

```
B34 = ((t - t3)/(t6 - t3)) * B33 + ((t7 - t)/(t7 - t4)) * B43;
B44 = ((t - t4)/(t7 - t4)) * B43 + ((t8 - t)/(t8 - t5)) * B53;
B54 = ((t - t5)/(t8 - t5)) * B53 + 0;

% Segment A
B02A = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B12A = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B22A = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B32A = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B42A = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B52A = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B62A = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B72A = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B03A = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B13A = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B23A = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B33A = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B43A = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B53A = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B63A = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B04A = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B14A = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B24A = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B34A = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B44A = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});
B54A = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {1,0,0,0,0,0,0,0});

% Segment B
B02B = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B12B = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B22B = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B32B = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B42B = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B52B = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B62B = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B72B = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B03B = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B13B = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B23B = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B33B = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B43B = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B53B = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B63B = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B04B = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B14B = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B24B = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B34B = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B44B = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});
B54B = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,1,0,0,0,0,0,0});

% Segment C
B02C = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B12C = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B22C = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
```

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B32C = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B42C = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B52C = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B62C = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B72C = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B03C = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B13C = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B23C = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B33C = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B43C = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B53C = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B63C = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B04C = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B14C = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B24C = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B34C = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B44C = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});
B54C = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,1,0,0,0,0,0});

% Segment D
B02D = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B12D = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B22D = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B32D = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B42D = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B52D = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B62D = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B72D = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B03D = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B13D = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B23D = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B33D = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B43D = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B53D = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B63D = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B04D = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B14D = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B24D = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B34D = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B44D = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});
B54D = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,1,0,0,0,0});

% Segment E
B02E = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B12E = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B22E = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B32E = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B42E = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B52E = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B62E = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B72E = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B03E = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B13E = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B23E = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
```

```
B33E = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B43E = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B53E = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B63E = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B04E = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B14E = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B24E = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B34E = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B44E = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});
B54E = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,1,0,0,0});

% Segment F
B02F = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B12F = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B22F = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B32F = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B42F = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B52F = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B62F = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B72F = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B03F = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B13F = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B23F = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B33F = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B43F = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B53F = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B63F = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B04F = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B14F = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B24F = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B34F = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B44F = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});
B54F = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,1,0,0});

% Segment G
B02G = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B12G = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B22G = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B32G = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B42G = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B52G = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B62G = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B72G = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B03G = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B13G = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B23G = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B33G = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B43G = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B53G = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B63G = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B04G = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B14G = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B24G = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B34G = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
```

```
B44G = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});
B54G = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,1,0});

% Segment H
B02H = subs(B02, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B12H = subs(B12, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B22H = subs(B22, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B32H = subs(B32, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B42H = subs(B42, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B52H = subs(B52, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B62H = subs(B62, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B72H = subs(B72, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B03H = subs(B03, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B13H = subs(B13, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B23H = subs(B23, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B33H = subs(B33, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B43H = subs(B43, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B53H = subs(B53, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B63H = subs(B63, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B04H = subs(B04, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B14H = subs(B14, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B24H = subs(B24, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B34H = subs(B34, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B44H = subs(B44, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
B54H = subs(B54, {B01,B11,B21,B31,B41,B51,B61,B71}, {0,0,0,0,0,0,0,1});
fprintf('Blending functions :\n');
B04 = [B04A, B04B, B04C, B04D, B04E, B04F, B04G, B04H]; B04 = simplify(B04)
B14 = [B14A, B14B, B14C, B14D, B14E, B14F, B14G, B14H]; B14 = simplify(B14)
B24 = [B24A, B24B, B24C, B24D, B24E, B24F, B24G, B24H]; B24 = simplify(B24)
B34 = [B34A, B34B, B34C, B34D, B34E, B34F, B34G, B34H]; B34 = simplify(B34)
B44 = [B44A, B44B, B44C, B44D, B44E, B44F, B44G, B44H]; B44 = simplify(B44)
fprintf('\n');
fprintf('General Equation of Curve :\n');
P = P0 * B04 + P1 * B14 + P2 * B24 + P3 * B34 + P4 * B44
fprintf('\n');
fprintf('Actual Equation :\n');
x = subs(P, ([P0, P1, P2, P3, P4]), ([x0, x1, x2, x3, x4])); x = simplify(x)
y = subs(P, ([P0, P1, P2, P3, P4]), ([y0, y1, y2, y3, y4])); y = simplify(y)

% plotting BF
tta = linspace(t0, t1);
ttb = linspace(t1, t2);
ttc = linspace(t2, t3);
ttd = linspace(t3, t4);
tte = linspace(t4, t5);
ttf = linspace(t5, t6);
ttg = linspace(t6, t7);
tth = linspace(t7, t8);
B04aa = subs(B04A, t, tta);
B04bb = subs(B04B, t, ttb);
B04cc = subs(B04C, t, ttc);
B04dd = subs(B04D, t, ttd);
B04ee = subs(B04E, t, tte);
B04ff = subs(B04F, t, ttf);
```

```
B04gg = subs(B04G, t, ttg);
B04hh = subs(B04H, t, tth);
B14aa = subs(B14A, t, tta);
B14bb = subs(B14B, t, ttb);
B14cc = subs(B14C, t, ttc);
B14dd = subs(B14D, t, ttd);
B14ee = subs(B14E, t, tte);
B14ff = subs(B14F, t, ttf);
B14gg = subs(B14G, t, ttg);
B14hh = subs(B14H, t, tth);
B24aa = subs(B24A, t, tta);
B24bb = subs(B24B, t, ttb);
B24cc = subs(B24C, t, ttc);
B24dd = subs(B24D, t, ttd);
B24ee = subs(B24E, t, tte);
B24ff = subs(B24F, t, ttf);
B24gg = subs(B24G, t, ttg);
B24hh = subs(B24H, t, tth);
B34aa = subs(B34A, t, tta);
B34bb = subs(B34B, t, ttb);
B34cc = subs(B34C, t, ttc);
B34dd = subs(B34D, t, ttd);
B34ee = subs(B34E, t, tte);
B34ff = subs(B34F, t, ttf);
B34gg = subs(B34G, t, ttg);
B34hh = subs(B34H, t, tth);
B44aa = subs(B44A, t, tta);
B44bb = subs(B44B, t, ttb);
B44cc = subs(B44C, t, ttc);
B44dd = subs(B44D, t, ttd);
B44ee = subs(B44E, t, tte);
B44ff = subs(B44F, t, ttf);
B44gg = subs(B44G, t, ttg);
B44hh = subs(B44H, t, tth);
figure,
plot(tta, B04aa, 'k-', ttb, B04bb, 'k--', ttc, B04cc, 'k-.', ttd, ...
B04dd, 'b-', tte, B04ee, 'b--', ttf, B04ff, 'b-.', ttg, B04gg, 'r-', tth, B04hh, 'r--');
hold on;
plot(tta, B14aa, 'k-', ttb, B14bb, 'k--', ttc, B14cc, 'k-.', ttd, ...
B14dd, 'b-', tte, B14ee, 'b--', ttf, B14ff, 'b-.', ttg, B14gg, 'r-', tth, B14hh, 'r--');
plot(tta, B24aa, 'k-', ttb, B24bb, 'k--', ttc, B24cc, 'k-.', ttd, ...
B24dd, 'b-', tte, B24ee, 'b--', ttf, B24ff, 'b-.', ttg, B24gg, 'r-', tth, B24hh, 'r--');
plot(tta, B34aa, 'k-', ttb, B34bb, 'k--', ttc, B34cc, 'k-.', ttd, ...
B34dd, 'b-', tte, B34ee, 'b--', ttf, B34ff, 'b-.', ttg, B34gg, 'r-', tth, B34hh, 'r--');
plot(tta, B44aa, 'k-', ttb, B44bb, 'k--', ttc, B44cc, 'k-.', ttd, ...
B44dd, 'b-', tte, B44ee, 'b--', ttf, B44ff, 'b-.', ttg, B44gg, 'r-', tth, B44hh, 'r--');
xlabel ('t'); ylabel('B'); title('B04 - B14 - B24 - B34 - B44');
legend('A', 'B', 'C', 'D', 'E', 'F', 'G', 'H');
hold off;

%plotting curve
xa = x(1); ya = y(1);
xb = x(2); yb = y(2);
xc = x(3); yc = y(3);
```

```
xd = x(4); yd = y(4);
xe = x(5); ye = y(5);
xf = x(6); yf = y(6);
xg = x(7); yg = y(7);
xh = x(8); yh = y(8);
xaa = subs(xa, t, tta); yaa = subs(ya, t, tta);
xbb = subs(xb, t, ttb); ybb = subs(yb, t, ttb);
xcc = subs(xc, t, ttc); ycc = subs(yc, t, ttc);
xdd = subs(xd, t, ttd); ydd = subs(yd, t, ttd);
xee = subs(xe, t, tte); yee = subs(ye, t, tte);
xff = subs(xf, t, ttf); yff = subs(yf, t, ttf);
xgg = subs(xg, t, ttg); ygg = subs(yg, t, ttg);
xhh = subs(xh, t, tth); yhh = subs(yh, t, tth);
X = [x0, x1, x2, x3, x4]; Y = [y0, y1, y2, y3, y4];
figure
subplot(131), plot(tta, xaa, 'k-', ttb, xbb, 'k--', ttc, xcc, 'k-.', ...
ttd, xdd, 'b-', tte, xee, 'b--', ttf, xff, 'b-.', ttg, xgg, 'r-', tth, xhh, 'r--');
xlabel('t'); ylabel('x'); title('t-x'); axis square;
subplot(132), plot(tta, yaa, 'k-', ttb, ybb, 'k--', ttc, ycc, 'k-.', ...
ttd, ydd, 'b-', tte, yee, 'b--', ttf, yff, 'b-.', ttg, ygg, 'r-', tth, yhh, 'r--');
xlabel('t'); ylabel('y'); title('t-y'); axis square;
subplot(133), plot(xaa, yaa, 'k-', xbb, ybb, 'k--', xcc, ycc, 'k-.', ...
xdd, ydd, 'b-', xee, yee, 'b--', xff, yff, 'b-.', xgg, ygg, 'r-', xhh, yhh, 'r--');
hold on;
scatter(X, Y, 20, 'r', 'filled');
xlabel('x'); ylabel('y'); title('x-y'); axis square; grid;
axis([-1.5 1.5 -1.5 1.5]);
hold off;
```

开放均匀样条和非均匀样条的概念与前面讨论的二次样条的概念相似，请读者自己扩展三次样条的概念。

3.9 本章小结

以下几点总结了本章讨论的主题：

- 近似样条一般不通过它们的任何 CP。
- B 样条是为克服贝塞尔样条的缺点而提出的近似样条。
- B 样条由多个在连接点处具有连续性的曲线段组成。
- 连接点处的参数变量 t 的值存储在 KV 中。
- 均匀 B 样条在 KV 中具有均匀的间隙。
- B 样条的 BF 使用 Cox de Boor 算法计算。
- B 样条有两个定义参数，d 与它的阶数有关，n 与 CP 的数量有关。
- CP 的数量可以独立于 B 样条的阶数进行更改。
- BF 和 B 样条方程由于分段而由多个部分组成。
- 均匀 B 样条的 BF 具有对称的形状，但相互偏移。
- 更改 CP 仅影响特定段而不是整个曲线。
- 空间 B 样条曲线与 KV 值无关。
- 开放均匀 B 样条具有重复值的 KV，称为多重性。

- 多重性迫使逼近 B 样条曲线表现得像混合样条曲线。
- 不均匀的 B 样条在 KV 中具有不均匀的间距。
- 非均匀 B 样条的 BF 在形状上是不对称的。

3.10 复习题

1. B 样条和贝塞尔样条的主要区别是什么?
2. 区分均匀 B 样条、开放均匀 B 样条和非均匀 B 样条。
3. 什么是 B 样条曲线的 KV?
4. 如何使用 Cox de Boor 算法计算 B 样条的 BF?
5. B 样条在什么条件下可以表现得像混合样条?
6. B 样条的局部控制属性是什么意思?
7. CP 的数量是否可以独立于 B 样条的次数而改变?
8. 改变 KV 对空间 B 样条曲线有何影响?
9. 为什么一个 B 样条曲线方程有多个子分量?
10. 什么是 KV 的多重性,它如何影响 BF 和样条曲线?

3.11 练习题

1. 找出与 CP,即(1,0)、(−1,1)和(1,−1)相关的线性均匀 B 样条方程。

2. 导出 $d=2$ 和 $n=3$ 的均匀线性 B 样条的 BF。

3. 导出与 5 个 CP 相关的二次均匀 B 样条的 BF。

4. 二次的 B 样条与 4 个 CP 相关联。若 KV 的形式为 $\boldsymbol{T}=[0,0.2,0.5,0.7,\cdots]$,请使用 Cox de Boor 算法为前两个曲线段 A 和 B 找到第一个 BF,即 B_{03} 的表达式。

5. 一阶均匀 B 样条与 3 个 CP,即 P_0、P_1 和 P_2 相关联。若 KV 的形式为 $\boldsymbol{T}=[0,0.4,0.5,0.8,\cdots]$,请使用 Cox de Boor 算法导出第二条曲线段的方程。

6. 非均匀 B 样条的阶数为 1,并与 3 个 CP,即 P_0、P_1 和 P_2 相关联。若 KV 为 $\boldsymbol{T}=[0,4,5,8,9]$,请导出第一和第二曲线段的方程。

7. 找到具有 CP,即(2,5)、(4,−1)、(5,8)和(7,−5)的二次均匀 B 样条的方程。

8. 找到具有 CP,即(2,5)、(4,−1)、(5,8)和(7,−5)且 KV 为 $\boldsymbol{T}=[0,4、5、8、9、13、15]$ 的二次非均匀 B 样条的方程。

9. 找到具有 CP,即(2,0)、(4,1)、(5,7)、(6,−5)和(8,−1)的三次均匀 B 样条的前两段的方程。

10. 找到 $d=4$ 和 $n=4$ 且 KV 为 $\boldsymbol{T}=[0,1,2,5,6,8,10,12,13]$ 的三次非均匀 B 样条的前两个 BF。