CHAPTER 3

SOLID-STATE DIODES AND DIODE CIRCUITS

第3章　固态二极管和二极管电路

本章提纲

3.1　*pn*结二极管
3.2　二极管的*i*-*v*特性
3.3　二极管方程：二极管的数学模型
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本章目标

- 理解二极管结构及基本版图；
- 了解*pn*结的电学特性；
- 熟悉各种二极管模型，包括数学模型、理想模型及恒定压降模型；
- 理解二极管的SPICE描述及二极管的模型参数；
- 熟悉二极管的工作区，包括正向偏置、反向偏置以及反向击穿；
- 能在电路分析中应用不同模型；
- 熟悉不同类型二极管，包括齐纳二极管、变容二极管、肖特基势垒二极管、太阳能电池和发光二极管（LED）；
- 熟悉 *pn* 结二极管的动态开关行为；

- 熟悉二极管整流器；
- 熟悉二极管电路SPICE。

本章导读

本章主要研究了固态二极管相关的原理、特性、模型及电路应用。首先从*pn*结入手，介绍*pn*结的电学特性。*pn*结二极管是*p*型半导体和*n*型半导体的紧密接触形成的。在*pn*结二极管中，冶金结附近存在较大的浓度梯度，由此产生较大的电子和空穴扩散电流。在零偏条件下，二极管两端以及空间电荷区中均没有电流存在。空间电荷区将产生内建电势和内部电场，而内部电场将导致电子和空穴电流的漂移电流，正好与扩散电流相抵消。

接下来，研究了二极管两端施加电压时的*i*-*v*特性以及二极管模型方程。在二极管两端施加电压时，二极管内部结区域的平衡遭到破坏，产生电流的传导。根据二极管的*i*-*v*特性可以通过二极管方程实现精确建模。根据二极管两端加载电压的方向不同，分为二极管反偏和正偏。反偏条件下二极管电流等于$-I_S$，这个值非常小。正偏条件下，电流可以很大，二极管的压降为0.6~0.7V。室温下，二极管电压每变化60mV可以引起10倍的二极管电流变化；室温下，硅二极管的温度系数为−1.8mV/℃。

击穿现象是二极管电路中的常见现象。本章对击穿现象的定义、预防及利用都进行了介绍。如果二极管两端的反向偏压过大，内部电场会使二极管发生击穿，包括齐纳击穿和雪崩击穿。工作在击穿区的二极管具有基本固定的压降，必须严格限制二极管上的电流，否则很容易烧毁器件。工作在击穿区的齐纳二极管可用于稳压器电路设计。电压调整率和负载调整率分别用于表征输出电压随输入电压和输出电流的变化。

二极管电容对于二极管的性能具有很大的影响。本章给出了二极管电容产生的原因以及影响。如果二极管两端电压发生变化，则在空间电荷区附近存储的电荷量也会随之变化，这意味着二极管模型中应包含电容。二极管反向偏置时，电容与外加电压的平方根成反比；正向偏置时，电容与工作电流和二极管传输时间成正比。由于这一电容的存在，使得二极管不能立即截止或导通，在截止时会形成一个电荷存储延迟。

本章重点给出了二极管电路计算和模拟的主要方法，包括迭代法、负载线分析法、恒压降法、SPICE等。在电路计算中，迭代法主要针对直接使用二极管方程时的情况。负载线分析法、理想二极管模型以及恒压降法常被用于二极管电路的简化分析。SPICE电路分析程序包含了精确描述理想和非理想二极管特性的内建模型，能够方便地分析含有二极管的电路特性。

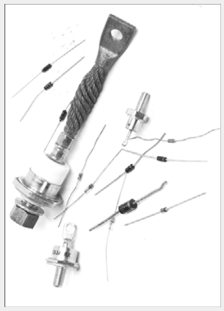
在二极管的应用方面，本章重点讲解了半波、全波以及全波桥式整流电路。整流电路主要将交流电压转换为直流电压。在电源电路中使用的整流器必须能够经受大的周期峰值电流，以及刚上电时的浪涌电流。在整流电路中，滤波电容的设计将决定纹波电压和二极管的导通角。由于存在内部电容，二极管的导通和截止必须经过电容充放电，因而不能瞬间完成。导通时间通常都是很小的，但截止过程则要慢得多，它必须将二极管中的存储电荷都转移掉，这样就产生一个存储延迟τ_S。在存储延迟期间，可能出现较大的反向电流。

本章最后研究了*pn*结发光和检测光的能力，讨论了光电二极管、太阳能电池和发光二极管的基本特性。

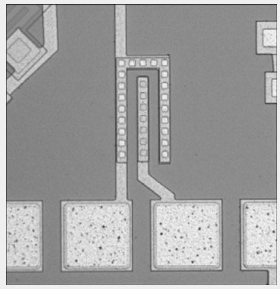
CHAPTER OUTLINE

CHAPTER GOALS

- Understand diode structure and basic layout
- Develop electrostatics of the *pn* junction
- Explore various diode models including the mathematical model, the ideal diode model, and the constant voltage drop model
- Understand the SPICE representation and model parameters for the diode
- Define regions of operation of the diode, including forward and reverse bias and reverse breakdown
- Apply the various types of models in circuit analysis
- Explore different types of diodes including Zener, variable capacitance, and Schottky barrier diodes as well as solar cells and light emitting diodes (LEDs)
- Discuss the dynamic switching behavior of the *pn* junction diode
- Explore diode rectifiers
- Practice simulating diode circuits using SPICE



Photograph of an assortment of diodes



Fabricated diode

The first electronic circuit element that we explore is the solid-state *pn* junction diode. The diode is an extremely important device in its own right with many important applications including ac-dc power conversion (rectification), solar power generation, and high-frequency mixers for RF communications. In addition, the *pn* junction diode is a fundamental building block for other solid-state devices. In later chapters, we will find that two closely coupled diodes are used to form the bipolar junction transistor (BJT), and two diodes form an integral part of the metal-oxide-semiconductor field-effect transistor (MOSFET) and the junction field-effect transistor (JFET). Gaining an understanding of diode characteristics is prerequisite to understanding the behavior of the field-effect and bipolar transistors that are used to realize both digital logic circuits and analog amplifiers.

The *pn* junction diode is formed by fabricating adjoining regions of *p*-type and *n*-type semiconductor material. Another type of diode, called the Schottky barrier diode, is formed by a non-ohmic contact between a metal such as aluminum, palladium, or platinum and an *n*-type or *p*-type semiconductor. Both types of solid-state diodes are discussed in this chapter. The vacuum diode, which was used before the advent of semiconductor diodes, still finds application in very high voltage situations.

The *pn* junction diode is a nonlinear element, and for many of us, this will be our first encounter with a nonlinear device. The diode is a two-terminal circuit element similar to a resistor, but its *i-v* characteristic, the relationship between the current through the element and the voltage across the element, is not a straight line. This nonlinear behavior allows electronic circuits to be designed to provide many useful operations, including rectification, mixing (a form of multiplication), and wave shaping. Diodes can also be used to perform elementary logic operations such as the AND and OR functions.

This chapter begins with a basic discussion of the structure and behavior of the *pn* junction diode and its terminal characteristics. Next is an introduction to the concept of modeling, and several different models for the diode are introduced and used to analyze the behavior of diode circuits. We begin to develop the intuition needed to make choices between models of various complexities in order to simplify electronic circuit analysis and design. Diode circuits are then explored, including the detailed application of the diode in rectifier circuits. The characteristics of Zener diodes, photo diodes, solar cells, and light-emitting diodes are also discussed.

3.1 THE *pn* JUNCTION DIODE

The ***pn* junction diode** is formed by fabrication of a p-type semiconductor region in intimate contact with an n-type semiconductor region, as illustrated in Fig. 3.1. The diode is constructed using the impurity doping process discussed in the last section of Chapter 2.

An actual diode can be formed by starting with an n-type wafer with doping N_D and selectively converting a portion of the wafer to p-type by adding acceptor impurities with $N_A > N_D$. The point at which the material changes from p-type to n-type is called the metallurgical junction. The p-type region is also referred to as the **anode** of the diode, and the n-type region is called the **cathode** of the diode.

Figure 3.2 gives the circuit symbol for the diode, with the left-hand end corresponding to the p-type region of the diode and the right-hand side corresponding to the n-type region. We will see shortly that the “arrow” points in the direction of positive current in the diode.

3.1.1 *pn* JUNCTION ELECTROSTATICS

Consider a *pn* junction diode similar to Fig. 3.1 having $N_A = 10^{17}/\text{cm}^3$ on the p-type side and $N_D = 10^{16}/\text{cm}^3$ on the n-type side. The hole and electron concentrations on the two sides of the junction will be

$$\begin{array}{lll} p\text{-type side:} & p_p = 10^{17}\ \text{holes/cm}^3 & n_p = 10^{3}\ \text{electrons/cm}^3 \\ n\text{-type side:} & p_n = 10^{4}\ \text{holes/cm}^3 & n_n = 10^{16}\ \text{electrons/cm}^3 \end{array} \tag{3.1}$$

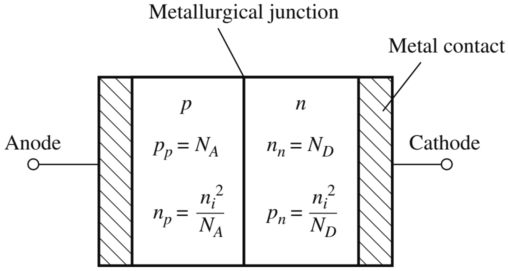


Figure 3.1 Basic *pn* junction diode.

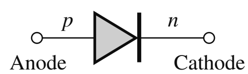


Figure 3.2 Diode circuit symbol.

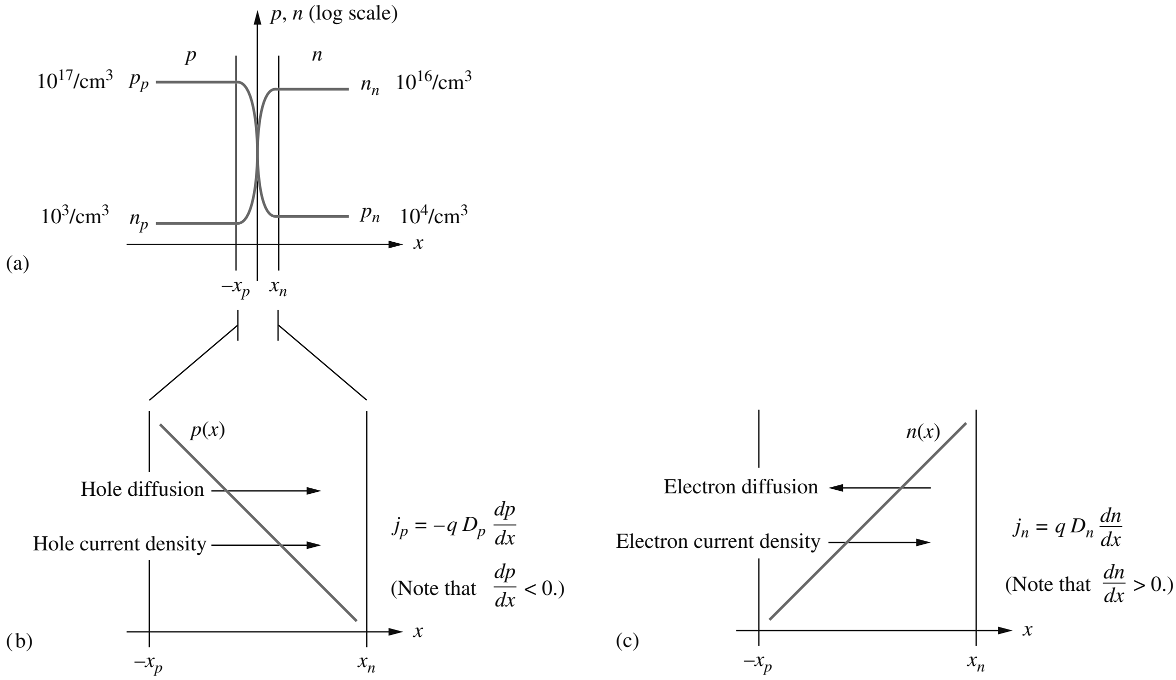


Figure 3.3 (a) Carrier concentrations; (b) hole diffusion current in the space charge region; (c) electron diffusion current in the space charge region.

As shown in Fig. 3.3(a), a very large concentration of holes exists on the *p*-type side of the metallurgical junction, whereas a much smaller hole concentration exists on the *n*-type side. Likewise, there is a very large concentration of electrons on the *n*-type side of the junction and a very low concentration on the *p*-type side.

From our knowledge of diffusion from Chapter 2, we know that mobile holes will diffuse from the region of high concentration on the *p*-type side toward the region of low concentration on the *n*-type side and that mobile electrons will diffuse from the *n*-type side to the *p*-type side, as in Fig. 3.3(b) and (c). If the diffusion processes were to continue unabated, there would eventually be a uniform concentration of holes and electrons throughout the entire semiconductor region, and the *pn* junction would cease to exist. Note that the two diffusion current densities are both directed in the positive *x* direction, but this is inconsistent with zero current in the open-circuited terminals of the diode.

A second, competing process must be established to balance the diffusion current. The competing mechanism is a drift current, as discussed in Chapter 2, and its origin can be understood by focusing on the region in the vicinity of the **metallurgical junction** shown in Fig. 3.4. As mobile holes move out of the *p*-type material, they leave behind immobile negatively charged acceptor atoms. Correspondingly, mobile electrons leave behind immobile ionized donor atoms with a localized positive charge. A **space charge region (SCR),** depleted of mobile carriers, develops in the region immediately around the metallurgical junction. This region is also often called the **depletion region,** or **depletion layer.**

From electromagnetics, we know that a region of space charge ρ_c (C/cm^3) will be accompanied by an electric field E measured in V/cm through Gauss' law

$$\nabla \cdot E = \frac{\rho_c}{\varepsilon_s} \tag{3.2}$$

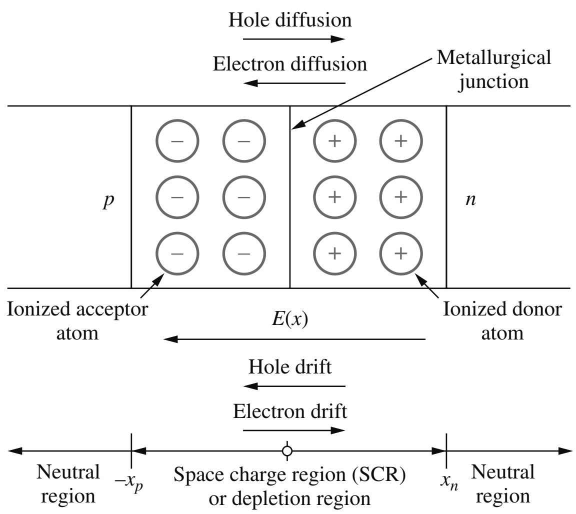


Figure 3.4 Space charge region formation near the metallurgical junction.

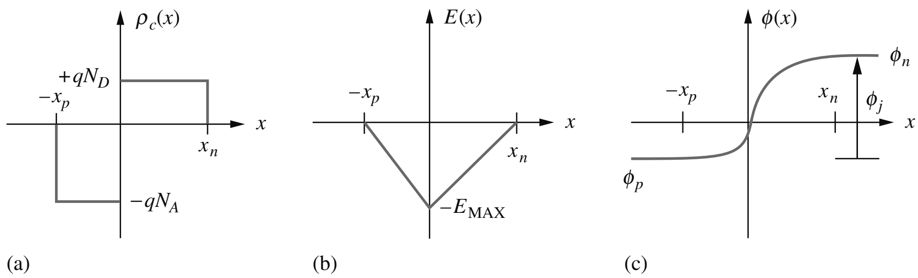


Figure 3.5 (a) Charge density (C/cm^3), (b) electric field (V/cm), and (c) electrostatic potential (V) in the space charge region of a *pn* junction.

written assuming a constant semiconductor permittivity ε_s (F/cm). In one dimension, Eq. (3.2) can be rearranged to give

$$E(x) = \frac{1}{\varepsilon_s} \int \rho_c(x)\, dx \tag{3.3}$$

Figure 3.5 illustrates the space charge and electric field in the diode for the case of uniform (constant) doping on both sides of the junction. As illustrated in Fig. 3.5(a), the value of the space charge density on the p-type side will be $-qN_A$ and will extend from the metallurgical junction at $x = 0$ to $-x_p$, whereas that on the n-type side will be $+qN_D$ and will extend from 0 to $+x_n$. The overall diode must be charge neutral, so

$$qN_A x_p = qN_D x_n \tag{3.4}$$

The electric field is proportional to the integral of the space charge density and will be zero in the (charge) neutral regions outside of the depletion region. Using this zero-field boundary condition yields the triangular electric field distribution in Fig. 3.5(b).

Figure 3.5(c) represents the integral of the electric field and shows that a **built-in potential** or **junction potential** ϕ_j exists across the pn junction space charge region according to

$$\phi_j = -\int E(x)\, dx \qquad \text{V} \tag{3.5}$$

ϕ_j represents the difference in the internal chemical potentials between the n and p sides of the diode, and it can be shown [1] to be given by

$$\phi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) \tag{3.6}$$

where the **thermal voltage** $V_T = kT/q$ was originally defined in Chapter 2.

Equations (3.3) to (3.6) can be used to determine the total width of the depletion region w_{do} in terms of the built-in potential:

$$w_{do} = (x_n + x_p) = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j} \quad \text{m} \tag{3.7}$$

From Eq. (3.7), we see that the doping on the more lightly doped side of the junction will be the most important in determining the **depletion-layer width.**

EXAMPLE 3.1 DIODE SPACE CHARGE REGION WIDTH

When diodes are actually fabricated, the doping levels on opposite sides of the pn junction tend to be quite asymmetric, and the resulting depletion layer tends to extend primarily on one side of the junction and is referred to as a "one-sided" step junction or one-sided abrupt junction. The pn junction that we analyze provides an example of the magnitudes of the distances involved in such a pn junction.

PROBLEM Calculate the built-in potential and depletion-region width for a silicon diode with $N_A = 10^{17}/\text{cm}^3$ on the p-type side and $N_D = 10^{20}/\text{cm}^3$ on the n-type side.

SOLUTION **Known Information and Given Data:** On the p-type side, $N_A = 10^{17}/\text{cm}^3$; on the n-type side, $N_D = 10^{20}/\text{cm}^3$. Theory describing the pn junction is given by Eqs. (3.4) through (3.7).

Unknowns: Built-in potential ϕ_j and depletion-region width w_{do}

Approach: Find the built-in potential using Eq. (3.6); use ϕ_j to calculate w_{do} in Eq. (3.7).

Assumptions: The diode operates at room temperature with $V_T = 0.025$ V. There are only donor impurities on the n-type side and acceptor impurities on the p-type side of the junction. The doping levels are constant on each side of the junction.

Analysis: The built-in potential is given by

$$\phi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) = (0.025\text{ V}) \ln\left[\frac{(10^{17}/\text{cm}^3)(10^{20}/\text{cm}^3)}{(10^{20}/\text{cm}^6)}\right] = 0.979\text{ V}$$

For silicon, $\varepsilon_s = 11.7\varepsilon_o$, where $\varepsilon_o = 8.85 \times 10^{-14}$ F/cm represents the permittivity of free space.

$$w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j}$$

$$w_{do} = \sqrt{\frac{2 \cdot 11.7 \cdot (8.85 \times 10^{-14}\text{ F/cm})}{1.60 \times 10^{-19}\text{ C}}\left(\frac{1}{10^{17}/\text{cm}^3} + \frac{1}{10^{20}/\text{cm}^3}\right) 0.979\text{ V}} = 0.113\ \mu\text{m}$$

Check of Results: The built-in potential should be less than the bandgap of the material. For silicon the bandgap is approximately 1.12 V (see Table 2.3), so ϕ_j appears reasonable. The depletion-layer width seems quite small, but a double check of the numbers indicates that the calculation is correct.

Discussion: The numbers in this example are fairly typical of a *pn* junction diode. For the normal doping levels encountered in solid-state diodes, the built-in potential ranges between 0.5 V and 1.0 V, and the total depletion-layer width w_{do} can range from a fraction of 1 μm in heavily doped diodes to tens of microns in lightly doped diodes.

EXERCISE: Calculate the built-in potential and depletion-region width for a silicon diode if N_A is increased to $2 \times 10^{18}/\text{cm}^3$ on the *p*-type side and $N_D = 10^{20}/\text{cm}^3$ on the *n*-type side.

ANSWERS: 1.05 V; 0.0263 μm

3.1.2 INTERNAL DIODE CURRENTS

Remember that the electric field E points in the direction that a positive carrier will move, so electrons drift toward the positive x direction and holes drift in the negative x direction in Fig. 3.4. Because the terminal currents must be zero, a dynamic equilibrium is established in the junction region. Hole diffusion is precisely balanced by hole drift, and electron diffusion is exactly balanced by electron drift. This balance is stated mathematically in Eq. (3.8), in which the total hole and electron current densities must each be identically zero:

$$j_n^T = qn\mu_n E + qD_n \frac{\partial n}{\partial x} = 0 \quad \text{and} \quad j_p^T = qp\mu_p E - qD_p \frac{\partial p}{\partial x} = 0 \qquad \text{A/cm}^2 \tag{3.8}$$

The difference in potential in Fig. 3.5(c) represents a barrier to both hole and electron flow across the junction. When a voltage is applied to the diode, the potential barrier is modified, and the delicate balances in Eq. (3.8) are disturbed, resulting in a current in the diode terminals.

EXAMPLE 3.2 DIODE ELECTRIC FIELD AND SPACE-CHARGE REGION EXTENTS

Now we find the value of the electric field in the diode and the size of the individual depletion layers on either side of the *pn* junction.

PROBLEM Find x_n, x_p, and E_{MAX} for the diode in Ex. 3.1.

SOLUTION **Known Information and Given Data:** On the *p*-type side, $N_A = 10^{17}/\text{cm}^3$; on the *n*-type side, $N_D = 10^{20}/\text{cm}^3$. Theory describing the *pn* junction is given by Eqs. (3.4) through (3.7). From Ex. 3.1, $\phi_j = 0.979$ V and $w_{do} = 0.113$ μm.

Unknowns: x_n, x_p, and E_{MAX}

Approach: Use Eqs. (3.4) and (3.7) to find x_n and x_p; use Eq. (3.5) to find E_{MAX}.

Assumptions: Room temperature operation

Analysis: Using Eq. (3.4), we can write

$$w_{do} = x_n + x_p = x_n\left(1 + \frac{N_D}{N_A}\right) \qquad \text{and} \qquad w_{do} = x_n + x_p = x_p\left(1 + \frac{N_A}{N_D}\right)$$

Solving for x_n and x_p gives

$$x_n = \frac{w_{do}}{\left(1+\dfrac{N_D}{N_A}\right)} = \frac{0.113\ \mu\text{m}}{\left(1+\dfrac{10^{20}/\text{cm}^3}{10^{17}/\text{cm}^3}\right)} = 1.13\times10^{-4}\ \mu\text{m}$$

and

$$x_p = \frac{w_{do}}{\left(1+\dfrac{N_A}{N_D}\right)} = \frac{0.113\ \mu\text{m}}{\left(1+\dfrac{10^{17}/\text{cm}^3}{10^{20}/\text{cm}^3}\right)} = 0.113\ \mu\text{m}$$

Equation (3.5) indicates that the built-in potential is equal to the area under the triangle in Fig. 3.5(b). The height of the triangle is $(-E_{\text{MAX}})$ and the base of the triangle is $x_n + x_p = w_{do}$:

$$\phi_j = \frac{1}{2}E_{\text{MAX}}w_{do} \qquad \text{and} \qquad E_{\text{MAX}} = \frac{2\phi_j}{w_{do}} = \frac{2(0.979\ \text{V})}{0.113\ \mu\text{m}} = 173\ \text{kV/cm}$$

Check of Results: From Eqs. (3.3) and (3.4), E_{MAX} can also be found from the doping levels and depletion-layer widths on each side of the junction. The equation in the next exercise can be used as a check of the answer.

EXERCISE: Using Eq. (3.3) and Fig. 3.5(a) and (b), show that the maximum field is given by

$$E_{\text{MAX}} = \frac{qN_Ax_p}{\varepsilon_s} = \frac{qN_Dx_n}{\varepsilon_s}$$

Use this formula to find E_{MAX}.

ANSWER: 175 kV/cm

EXERCISE: Calculate E_{MAX}, x_p, and x_n for a silicon diode if $N_A = 2\times10^{18}/\text{cm}^3$ on the p-type side and $N_D = 10^{20}/\text{cm}^3$ on the n-type side. Use $\phi_j = 1.05$ V and $w_{do} = 0.0263\ \mu$m.

ANSWERS: 798 kV/cm; $5.16\times10^{-4}\ \mu$m; 0.0258 μm

3.2 THE *i-v* CHARACTERISTICS OF THE DIODE

The diode is the electronic equivalent of a mechanical check valve—it permits current to flow in one direction in a circuit, but prevents movement of current in the opposite direction. We will find that this nonlinear behavior has many useful applications in electronic circuit design. To understand this phenomenon, we explore the relationship between the current in the diode and the voltage applied to the diode. This information, called the i-v characteristic of the diode, is first presented graphically and then mathematically in this section and Sec. 3.3.

The current in the diode is determined by the voltage applied across the diode terminals, and the diode is shown with a voltage applied in Fig. 3.6. Voltage v_D represents the voltage applied between the diode terminals; i_D is the current through the diode. The neutral regions of the diode represent a low resistance to current, and essentially all the external applied voltage is dropped across the space charge region.

The applied voltage disturbs the balance between the drift and diffusion currents at the junction specified in the two expressions in Eq. (3.8). A positive applied voltage reduces the potential barrier for electrons and holes, as in Fig. 3.7, and current easily crosses the junction. A negative voltage

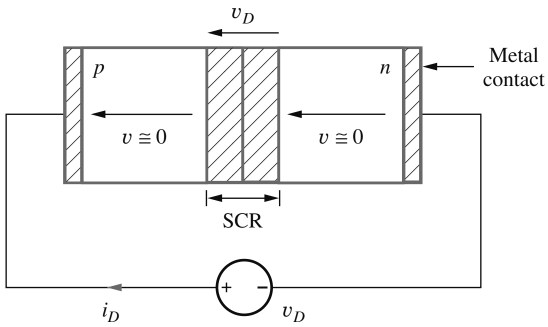


Figure 3.6 Diode with external applied voltage v_D.

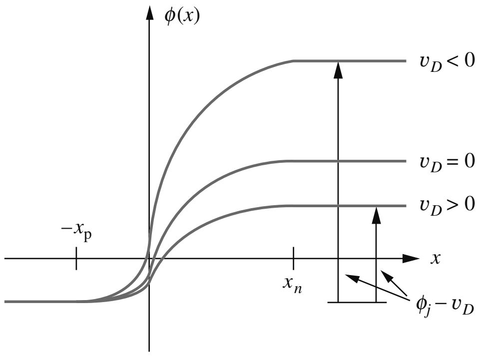


Figure 3.7 Electrostatic junction potential for different applied voltages.

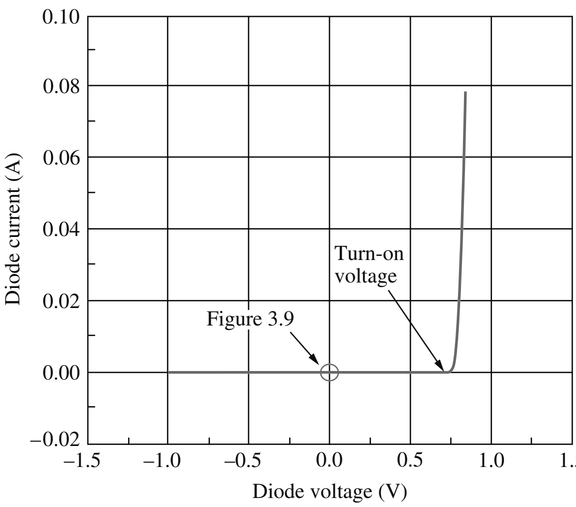


Figure 3.8 Graph of the *i-v* characteristics of a *pn* junction diode.

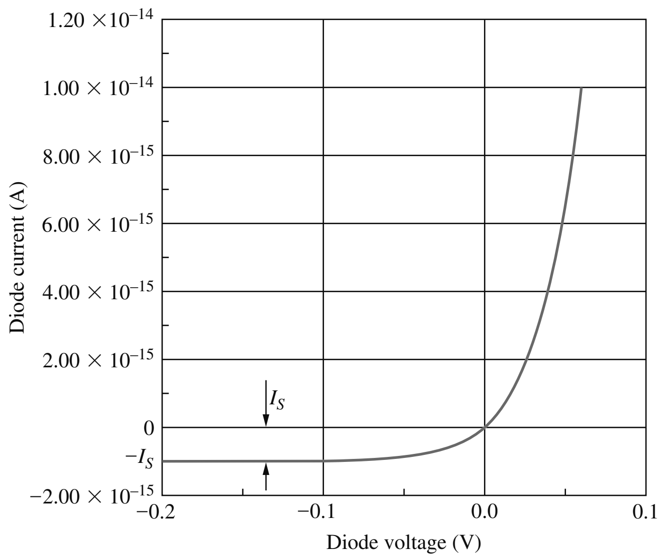


Figure 3.9 Diode behavior near the origin with $I_S = 10^{-15}$ A and $n = 1$.

increases the potential barrier, and although the balance in Eq. (3.8) is disturbed, the increased barrier results in a very small current.

The most important details of the diode *i-v* characteristic appear in Fig. 3.8. The diode characteristic is definitely not linear. For voltages less than zero, the diode is essentially nonconducting, with $i_D \cong 0$. As the voltage increases above zero, the current remains nearly zero until the voltage v_D exceeds approximately 0.5 to 0.7 V. At this point, the diode current increases rapidly, and the voltage across the diode becomes almost independent of current. The voltage required to bring the diode into significant conduction is often called either the **turn-on** or **cut-in voltage** of the diode.

Figure 3.9 is an enlargement of the region around the origin in Fig. 3.8. We see that the i-v characteristic passes through the origin; the current is zero when the applied voltage is zero. For negative voltages the current is not actually zero but reaches a limiting value labeled as $-I_S$ for voltages less than -0.1 V. $\boldsymbol{I_S}$ is called the **reverse saturation current,** or just **saturation current,** of the diode.

3.3 THE DIODE EQUATION: A MATHEMATICAL MODEL FOR THE DIODE

When performing both hand and computer analysis of circuits containing diodes, it is very helpful to have a mathematical representation, or model, for the i-v characteristics depicted in Figs. 3.8 and 3.9. In fact, solid-state device theory has been used to formulate a mathematical expression that agrees amazingly well with the measured i-v characteristics of the pn junction diode. We study this extremely important formula called the **diode equation** in this section.

A positive voltage v_D is applied to the diode in Fig. 3.10; in the figure the diode is represented by its circuit symbol from Fig. 3.2. Although we will not attempt to do so here, Eq. (3.8) can be solved for the hole and electron concentrations and the terminal current in the diode as a function of the voltage v_D across the diode. The resulting diode equation, given in Eq. (3.9), provides a **mathematical model** for the i-v characteristics of the diode:

$$i_D = I_S\left[\exp\left(\frac{qv_D}{nkT}\right) - 1\right] = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right] \tag{3.9}$$

where $I_S =$ reverse saturation current of diode (A)
$v_D =$ voltage applied to diode (V)
$q =$ electronic charge (1.60×10^{-19} C)
$k =$ Boltzmann's constant (1.38×10^{-23} J/K)
$T =$ absolute temperature (K)
$n =$ nonideality factor (dimensionless)
$V_T = kT/q =$ thermal voltage (V)

The total current through the diode is i_D, and the voltage drop across the diode terminals is v_D. Positive directions for the terminal voltage and current are indicated in Fig. 3.10. V_T is the thermal voltage encountered previously in Chapter 2 and will be assumed equal to 0.025 V at room temperature. I_S is the (reverse) saturation current of the diode encountered in Fig. 3.9, and n is a dimensionless parameter discussed in more detail shortly. The saturation current is typically in the range

$$10^{-18}\text{ A} \leq I_S \leq 10^{-9}\text{ A} \tag{3.10}$$

From device physics, it can be shown that the diode saturation current is proportional to n_i^2, where n_i is the density of electrons and holes in intrinsic semiconductor material. After reviewing Eq. (2.1) in Chapter 2, we realize that I_S will be strongly dependent on temperature. Detailed discussion of this temperature dependence is in Sec. 3.5.

Parameter n is termed the **nonideality factor.** For most silicon diodes, n is in the range 1.0 to 1.1, although it approaches a value of 2 in diodes operating at high current densities. From this point on, we assume that $n = 1$ unless otherwise indicated, and the diode equation will be written as

$$i_D = I_S\left[\exp\left(\frac{v_D}{V_T}\right) - 1\right] \tag{3.11}$$

It is difficult to distinguish small variations in the value of n from an uncertainty in our knowledge in

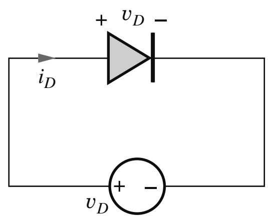


Figure 3.10 Diode with applied voltage v_D.

the absolute temperature. This is one reason that we will assume that $n = 1$ in this text. The problem can be investigated further by working on the next exercise.

EXERCISE: For $n = 1$ and $T = 300$ K, $n(KT/q) = 25.9$ mV. Verify this calculation. Now, suppose $n = 1.03$. What temperature gives the same value for nV_T?

ANSWER: 291 K

The mathematical model in Eq. (3.11) provides a highly accurate prediction of the *i*-*v* characteristics of the *pn* junction diode. The model is useful for understanding the detailed behavior of diodes and also forms the heart of the diode model in the SPICE circuit simulation program. It provides a basis for understanding the *i*-*v* characteristics of the bipolar transistor in Chapter 5.

DESIGN NOTE

The static *i*-*v* characteristics of the diode are well-characterized by three parameters: saturation current $\boldsymbol{I_S}$, temperature via the thermal voltage $\boldsymbol{V_T}$, and nonideality factor $\boldsymbol{n}$.

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right]$$

EXAMPLE 3.3 DIODE VOLTAGE AND CURRENT CALCULATIONS

In this example, we calculate some typical values of diode voltages for several different current levels and types of diodes.

PROBLEM (a) Find the diode voltage for a silicon diode with $I_S = 0.1$ fA operating at room temperature at a current of 300 μA. What is the diode voltage if $I_S = 10$ fA? What is the diode voltage if the current increases to 1 mA?
(b) Find the diode voltage for a silicon power diode with $I_S = 10$ nA and $n = 2$ operating at room temperature at a current of 10 A.
(c) A silicon diode is operating with a temperature of 50°C and the diode voltage is measured to be 0.736 V at a current of 2.50 mA. What is the saturation current of the diode?

SOLUTION (a) **Known Information and Given Data:** The diode currents are given and the saturation current parameter I_S is specified.

Unknowns: Diode voltage at each of the operating currents

Approach: Solve Eq. (3.9) for the diode voltage and evaluate the expression at each operating current.

Assumptions: At room temperature, we will use $V_T = 0.025 \text{ V} = 1/40 \text{ V}$; assume $n = 1$, since it is not specified otherwise; assume dc operation: $i_D = I_D$ and $v_D = V_D$.

Analysis: Solving Eq. (3.9) for V_D with $I_D = 0.1$ fA yields

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 1(0.025\text{ V})\ln\left(1 + \frac{3\times 10^{-4}\text{ A}}{10^{-16}\text{ A}}\right) = 0.718\text{ V}$$

For $I_S = 10$ fA:

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 1(0.025\text{ V})\ln\left(1 + \frac{3\times 10^{-4}\text{ A}}{10^{-14}\text{ A}}\right) = 0.603\text{ V}$$

For $I_D = 1$ mA with $I_S = 0.1$ fA:

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 1(0.025\text{ V})\ln\left(1 + \frac{10^{-3}\text{ A}}{10^{-16}\text{ A}}\right) = 0.748\text{ V}$$

Check of Results: The diode voltages are all between 0.5 V and 1.0 V and are reasonable (the diode voltage should not exceed the bandgap for $n = 1$).

SOLUTION (b) **Known Information and Given Data:** The diode current is given and the values of the saturation current parameter I_S and n are both specified.

Unknowns: Diode voltage at the operating current

Approach: Solve Eq. (3.9) for the diode voltage and evaluate the resulting expression.

Assumptions: At room temperature, we will use $V_T = 0.025\text{ V} = 1/40\text{ V}$.

Analysis: The diode voltage will be

$$V_D = nV_T \ln\left(1 + \frac{I_D}{I_S}\right) = 2(0.025\text{ V})\ln\left(1 + \frac{10\text{ A}}{10^{-8}\text{ A}}\right) = 1.04\text{ V}$$

Check of Results: Based on the comment at the end of part (a) and realizing that $n = 2$, voltages between 1 V and 2 V are reasonable for power diodes operating at high currents.

SOLUTION (c) **Known Information and Given Data:** The diode current is 2.50 mA and voltage is 0.736 V. The diode is operating at a temperature of 50°C.

Unknowns: Diode saturation current I_S

Approach: Solve Eq. (3.9) for the saturation current and evaluate the resulting expression. The value of the thermal voltage V_T will need to be calculated for $T = 50°\text{C}$.

Assumptions: The value of n is unspecified, so assume $n = 1$.

Analysis: Converting $T = 50°\text{C}$ to Kelvins, $T = (273 + 50)\text{ K} = 323\text{ K}$, and

$$V_T = \frac{kT}{q} = \frac{(1.38\times 10^{-23}\text{ J/K})(323\text{ K})}{1.60\times 10^{-19}\,°\text{C}} = 27.9\text{ mV}$$

Solving Eq. (3.9) for I_S yields

$$I_S = \frac{I_D}{\exp\left(\dfrac{V_D}{nV_T}\right) - 1} = \frac{2.5\text{ mA}}{\exp\left(\dfrac{0.736\text{ V}}{0.0279\text{ V}}\right) - 1} = 8.74\times 10^{-15}\text{ A} = 8.74\text{ fA}$$

Check of Results: The saturation current is within the range of typical values specified in Eq. (3.10).

EXERCISE: A diode has a reverse saturation current of 40 fA. Calculate i_D for diode voltages of 0.55 and 0.7 V. What is the diode voltage if $i_D = 6$ mA?

ANSWERS: 143 μA; 57.9 mA; 0.643 V

3.4 DIODE CHARACTERISTICS UNDER REVERSE, ZERO, AND FORWARD BIAS

When a dc voltage or current is applied to an electronic device, we say that we are providing a dc bias or simply a **bias** to the device. As we develop our electronics expertise, choosing the bias will be important to all of the circuits that we analyze and design. We will find that bias determines device characteristics, power dissipation, voltage and current limitations, and other important circuit parameters such as impedance levels and voltage gain. For a diode, there are three important bias conditions. **Reverse bias** and **forward bias** correspond to $v_D < 0$ V and $v_D > 0$ V, respectively. The **zero bias** condition, with $v_D = 0$ V, represents the boundary between the forward and reverse bias regions. When the diode is operating with reverse bias, we consider the diode "off" or nonconducting because the current is very small ($i_D = -I_S$). For forward bias, the diode is usually in a highly conducting state and is considered "on."

3.4.1 REVERSE BIAS

For $v_D < 0$, the diode is said to be operating under reverse bias. Only a very small reverse leakage current, approximately equal to I_S, flows through the diode. This current is small enough that we usually think of the diode as being in the nonconducting or off state when it is reverse-biased. For example, suppose that a dc voltage $V = -4V_T = -0.1$ V is applied to the diode terminals so that $v_D = -0.1$ V. Substituting this value into Eq. (3.11) gives

$$i_D = I_S\left[\exp\left(\frac{v_D}{V_T}\right) - 1\right] = I_S[\overbrace{\cancel{\exp(-4)}}^{\text{negligible}} - 1] \approx -I_S \tag{3.12}$$

because $\exp(-4) = 0.018$. For a reverse bias greater than $4V_T$, that is, $v_D \le -4V_T = -0.1$ V, the exponential term $\exp(v_D/V_T)$ is much less than 1, and the diode current will be approximately equal to $-I_S$, a very small current. The current I_S was identified in Fig. 3.9.

EXERCISE: A diode has a reverse saturation current of 5 fA. Calculate i_D for diode voltages of -0.04 V and -2 V (see Sec. 3.6).

ANSWERS: -3.99 fA; -5 fA

The situation depicted in Fig. 3.9 and Eq. (3.12) actually represents an idealized picture of the diode. In a real diode, the reverse leakage current is several orders of magnitude larger than I_S due to the generation of electron–hole pairs within the depletion region. In addition, i_D does not saturate but increases gradually with reverse bias as the width of the depletion layer increases with reverse bias. (See Sec. 3.6.1).

3.4.2 ZERO BIAS

Although it may seem to be a trivial result, it is important to remember that the i-v characteristic of the diode passes through the origin. For zero bias with $v_D = 0$, we find $i_D = 0$. Just as for a resistor, there must be a voltage across the diode terminals in order for a nonzero current to exist.

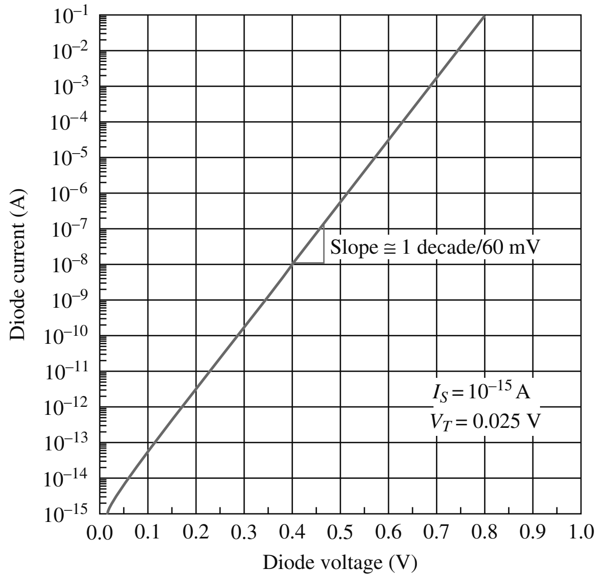


Figure 3.11 Diode *i*-*v* characteristic on semilog scale.

3.4.3 FORWARD BIAS

For the case $v_D > 0$, the diode is said to be operating under forward bias, and a large current can be present in the diode. Suppose that a voltage $v_D \geq +4V_T = +0.1$ V is applied to the diode terminals. The exponential term $\exp(v_D/V_T)$ is now much greater than 1, and Eq. (3.9) reduces to

$$i_D = I_S \left[\exp\left(\frac{v_D}{V_T}\right) - \overset{\text{negligible}}{\cancel{1}}\right] \cong I_S \exp\left(\frac{v_D}{V_T}\right) \tag{3.13}$$

The diode current grows exponentially with applied voltage for a forward bias greater than approximately $4V_T$.

The diode *i*-*v* characteristic for forward voltages is redrawn in semilogarithmic form in Fig. 3.11. The straight line behavior predicted by Eq. (3.13) for voltages $v_D \geq 4V_T$ is apparent. A slight curvature can be observed near the origin, where the −1 term in Eq. (3.13) is no longer negligible. The slope of the graph in the exponential region is very important. Only a 60-mV increase in the forward voltage is required to increase the diode current by a factor of 10. This is the reason for the almost vertical increase in current noted in Fig. 3.8 for voltages above the turn-on voltage.

EXAMPLE 3.4 **DIODE VOLTAGE CHANGE VERSUS CURRENT**

The slope of the diode i-v characteristic is an important number for circuit designers to remember.

PROBLEM Use Eq. (3.13) to accurately calculate the voltage change required to increase the diode current by a factor of 10.

SOLUTION **Known Information and Given Data:** The current changes by a factor of 10.

Unknowns: The diode voltage change corresponding to a one decade change in current; the saturation current has not been given.

Approach: Form an expression for the ratio of two diode currents using the diode equation. The saturation current will cancel out and is not needed.

Assumptions: Room temperature operation with $V_T = 25.0$ mV; assume $I_D \gg I_S$.

Analysis: Let

$$i_{D1} = I_S \exp\left(\frac{v_{D1}}{V_T}\right) \qquad \text{and} \qquad i_{D2} = I_S \exp\left(\frac{v_{D2}}{V_T}\right)$$

Taking the ratio of the two currents and setting it equal to 10 yields

$$\frac{i_{D2}}{i_{D1}} = \exp\left(\frac{v_{D2} - v_{D1}}{V_T}\right) = \exp\left(\frac{\Delta v_D}{V_T}\right) = 10 \qquad \text{and} \qquad \Delta v_D = V_T \ln 10 = 2.3V_T$$

Therefore $\Delta V_D = 2.3V_T = 57.5$ mV (or approximately 60 mV) at room temperature.

Check of Results: The result is consistent with the logarithmic plot in Fig. 3.11. The diode voltage changes approximately 60 mV for each decade change in forward current.

EXERCISE: A diode has a saturation current of 2 fA. (a) What is the diode voltage at a diode current of 40 μA (assume $V_T = 25.0$ mV)? Repeat for a diode current of 400 μA. What is the difference in the two diode voltages? (b) Repeat for $V_T = 25.9$ mV.

ANSWERS: 0.593 V, 0.651 V, 57.6 mV; 0.614 V, 0.674 V, 59.6 mV

DESIGN NOTE

The diode voltage changes by approximately *60 mV per decade* change in diode current. Sixty mV/decade often plays an important role in our thinking about the design of circuits containing both diodes and bipolar transistors and is a good number to remember.

Figure 3.12 compares the characteristics of three diodes with different values of saturation current. The saturation current of diode A is 10 times larger than that of diode B, and the saturation current of diode B is 10 times that of diode C. The spacing between each pair of curves is

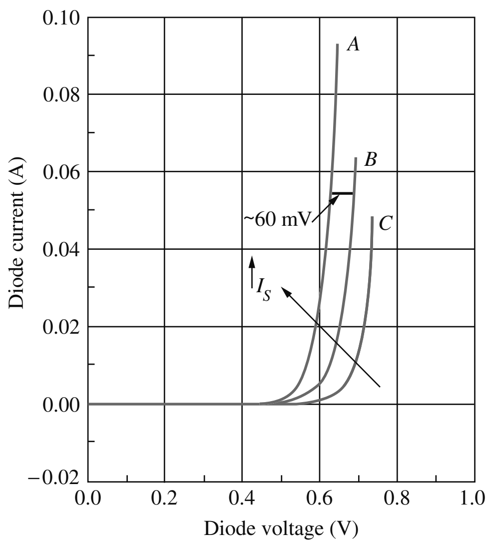


Figure 3.12 Diode characteristics for three different reverse saturation currents: (a) 10^{-12} A, (b) 10^{-13} A, and (c) 10^{-14} A.

approximately 60 mV. If the saturation current of the diode is reduced by a factor of 10, then the diode voltage must increase by approximately 60 mV to reach the same operating current level. Figure 3.12 also shows the relatively low sensitivity of the forward diode voltage to changes in the parameter I_S. For a fixed diode current, a change of two orders of magnitude in I_S results in a diode voltage change of only 120 mV.

3.5 DIODE TEMPERATURE COEFFICIENT

Another important number to keep in mind is the temperature coefficient associated with the diode voltage v_D. Solving Eq. (3.11) for the diode voltage under forward bias

$$v_D = V_T \ln\left(\frac{i_D}{I_S} + 1\right) = \frac{kT}{q} \ln\left(\frac{i_D}{I_S} + 1\right) \cong \frac{kT}{q} \ln\left(\frac{i_D}{I_S}\right) \qquad \text{V} \tag{3.14}$$

and taking the derivative with respect to temperature yields

$$\frac{dv_D}{dT} = \frac{k}{q} \ln\left(\frac{i_D}{I_S}\right) - \frac{kT}{q}\frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_D}{T} - V_T \frac{1}{I_S}\frac{dI_S}{dT} = \frac{v_D - V_{GO} - 3V_T}{T} \qquad \text{V/K} \tag{3.15}$$

where it is assumed that $i_D \gg I_S$ and $I_S \propto n_i^2$. In the numerator of Eq. (3.15), v_D represents the diode voltage, V_{GO} is the voltage corresponding to the silicon bandgap energy at 0 K ($V_{GO} = E_G/q$), and V_T is the thermal potential. The last two terms result from the temperature dependence of n_i^2 as defined by Eq. (2.2). Evaluating the terms in Eq. (3.15) for a silicon diode with $v_D = 0.65$ V, $E_G = 1.12$ eV, and $V_T = 0.025$ V yields

$$\frac{dv_D}{dT} = \frac{(0.65 - 1.12 - 0.075)\ \text{V}}{300\ \text{K}} = -1.82\ \text{mV/K} \tag{3.16}$$

DESIGN NOTE

The forward voltage of the diode decreases as temperature increases, and the diode exhibits a temperature coefficient of approximately -1.8 mV/°C at room temperature.

EXERCISE: (a) Verify Eq. (3.15) using the expression for n_i^2 from Eq. (2.1). (b) A silicon diode is operating at $T = 300$ K, with $i_D = 1$ mA, and $v_D = 0.680$ V. Use the result from Eq. (3.16) to estimate the diode voltage at 275 K and at 350 K.

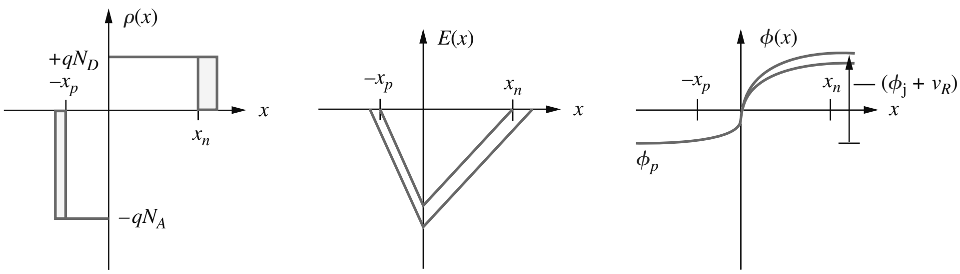
ANSWERS: 0.726 V; 0.589 V

3.6 DIODES UNDER REVERSE BIAS

We must be aware of several other phenomena that occur in diodes operated under reverse bias. As depicted in Fig. 3.13, the reverse voltage v_R applied across the diode terminals is dropped across the space charge region and adds directly to the built-in potential of the junction:

$$v_j = \phi_j + v_R \qquad \text{for } v_R > 0 \tag{3.17}$$

The increased voltage results in a larger internal electric field that must be supported by additional charge in the depletion layer, as defined by Eqs. (3.2) to (3.5). Using Eq. (3.7) with the voltage



(a) Space charge density (b) Electric field (c) Electrostatic potential

Figure 3.13 The *pn* junction diode under reverse bias.

from Eq. (3.17), the general expression for the depletion-layer width w_d for an applied reverse-bias voltage v_R becomes

$$w_d = (x_n + x_p) = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(\phi_j + v_R)}$$

or

$$w_d = w_{do}\sqrt{1 + \frac{v_R}{\phi_j}} \qquad \text{where } w_{do} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j} \tag{3.18}$$

The width of the space charge region increases approximately in proportion to the square root of the applied voltage.

EXERCISE: The diode in Ex. 3.1 had a zero-bias depletion-layer width of 0.113 μm and a built-in voltage of 0.979 V. What will be the depletion-layer width for a 10-V reverse bias? What is the new value of E_{MAX}?

ANSWERS: 0.378 μm; 581 kV/cm

3.6.1 SATURATION CURRENT IN REAL DIODES

The reverse saturation current actually results from the thermal generation of hole–electron pairs in the depletion region that surrounds the *pn* junction and is therefore proportional to the volume of the depletion region. Since the depletion-layer width increases with reverse bias, as described by Eq. (3.18), the reverse current does not truly saturate, as depicted in Fig. 3.9 and Eq. (3.9). Instead, there is gradual increase in reverse current as the magnitude of the reverse bias voltage is increased.

$$I_S = I_{SO}\sqrt{1 + \frac{v_R}{\phi_j}} \tag{3.19}$$

Under forward bias, the depletion-layer width changes very little, and $I_S = I_{SO}$ for forward bias.

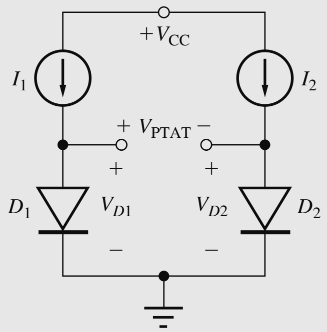
EXERCISE: A diode has $I_{SO} = 10$ fA and a built-in voltage of 0.8 V. What is I_S for a reverse bias of 10 V?

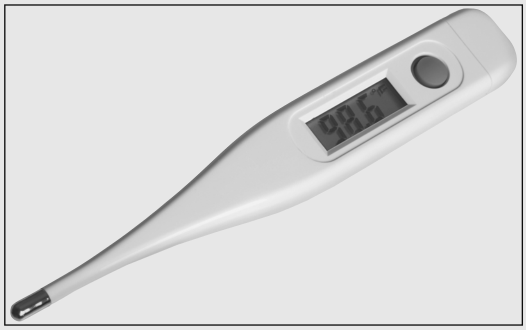
ANSWER: 36.7 fA

ELECTRONICS IN ACTION

The PTAT Voltage and Electronic Thermometry

The well-defined temperature dependence of the diode voltage discussed in Secs. 3.3 to 3.5 is actually used as the basis for most digital thermometers. We can build a simple electronic thermometer based on the circuit shown here in which two identical diodes are biased by current sources I_1 and I_2.





Digital thermometer: © *D. Hurst/Alamy RF*.

If we calculate the difference between the diode voltages using Eq. (3.14), we discover a voltage that is directly **proportional to absolute temperature** (PTAT), referred to as the PTAT voltage or V_{PTAT}:

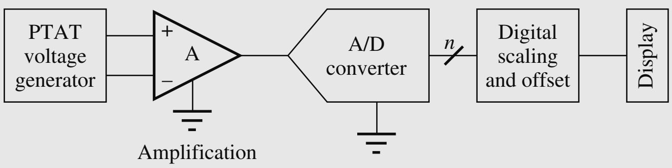
$$V_{\text{PTAT}} = V_{D1} - V_{D2} = V_T \ln\left(\frac{I_{D1}}{I_S}\right) - V_T \ln\left(\frac{I_{D2}}{I_S}\right) = V_T \ln\left(\frac{I_{D1}}{I_{D2}}\right) = \frac{kT}{q} \ln\left(\frac{I_{D1}}{I_{D2}}\right)$$

The PTAT voltage has a temperature coefficient given by

$$\frac{dV_{\text{PTAT}}}{dT} = \frac{k}{q} \ln\left(\frac{I_{D1}}{I_{D2}}\right) = \frac{V_{\text{PTAT}}}{T}$$

By using two diodes, the temperature dependence of I_S has been eliminated from the equation. For example, suppose $T = 295$ K, $I_{D1} = 250\ \mu\text{A}$, and $I_{D2} = 50\ \mu\text{A}$. Then $V_{\text{PTAT}} = 40.9$ mV with a temperature coefficient of $+0.139$ mV/K.

This simple but elegant PTAT voltage circuit forms the heart of most of today's highly accurate electronic thermometers as depicted in the block diagram here. The analog PTAT voltage is amplified and then converted to a digital representation by an A/D converter. The digital output is scaled and offset to properly represent either the Fahrenheit or Celsius temperature scales and appears on an alphanumeric display. The scaling and offset shift can also be done in analog form prior to the A/D conversion operation.



Block diagram of a digital thermometer.

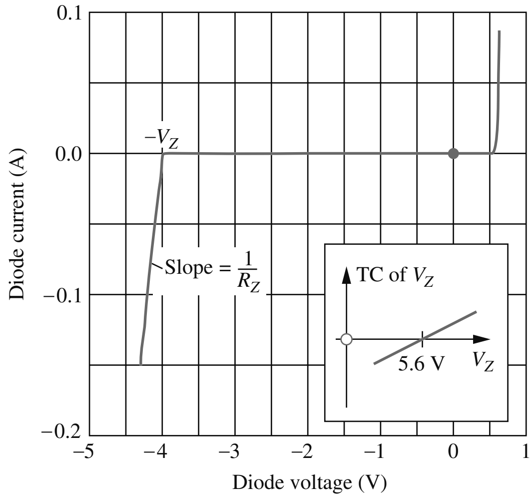


Figure 3.14 *i-v* characteristic of a diode including the reverse-breakdown region. The inset shows the temperature coefficient (TC) of V_Z.

3.6.2 REVERSE BREAKDOWN

As the reverse voltage increases, the electric field within the device grows, and the diode eventually enters the **breakdown region.** The onset of the breakdown process is fairly abrupt, and the current increases rapidly for any further increase in the applied voltage, as shown in the *i-v* characteristic of Fig. 3.14.

The magnitude of the voltage at which breakdown occurs is called the **breakdown voltage V_Z** of the diode and is typically in the range $2\text{ V} \leq V_Z \leq 2000\text{ V}$. The value of V_Z is determined primarily by the doping level on the more lightly doped side of the *pn* junction, but the heavier the doping, the smaller the breakdown voltage of the diode.

Two separate breakdown mechanisms have been identified: *avalanche breakdown* and *Zener breakdown.* These are discussed in the following two sections.

Avalanche Breakdown

Silicon diodes with breakdown voltages greater than approximately 5.6 V enter breakdown through a mechanism called **avalanche breakdown.** As the width of the depletion layer increases under reverse bias, the electric field increases, as indicated in Fig. 3.13. Free carriers in the depletion region are accelerated by this electric field, and as the carriers move through the depletion region, they collide with the fixed atoms. At some point, the electric field and the width of the space charge region become large enough that some carriers gain energy sufficient to break covalent bonds upon impact, thereby creating electron–hole pairs. The new carriers created can also accelerate and create additional electron–hole pairs through this **impact-ionization process,** as illustrated in Fig. 3.15.

Zener Breakdown

True **Zener breakdown** occurs only in heavily doped diodes. The high doping results in a very narrow depletion-region width, and application of a reverse bias causes carriers to tunnel directly between the conduction and valence bands, again resulting in a rapidly increasing reverse current in the diode.

Breakdown Voltage Temperature Coefficient

We can differentiate between the two types of breakdown because the breakdown voltages associated with the two mechanisms exhibit opposite temperature coefficients (TC). In avalanche breakdown,

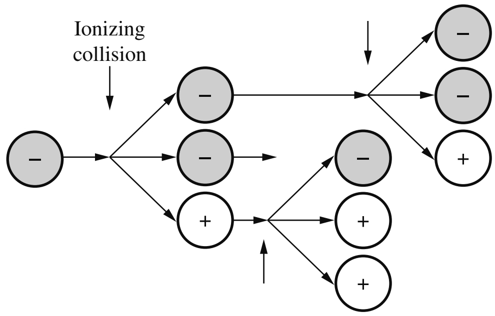


Figure 3.15 The avalanche breakdown process. (Note that the positive and negative charge carriers will actually be moving in opposite directions in the electric field in the depletion region.)

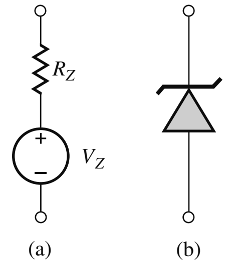


Figure 3.16 (a) Model for reverse-breakdown region of diode. (b) Zener diode symbol.

V_Z increases with temperature; in Zener breakdown, V_Z decreases with temperature. For silicon diodes, a zero temperature coefficient is achieved at approximately 5.6 V. The avalanche breakdown mechanism dominates in diodes that exhibit breakdown voltages of more than 5.6 V, whereas diodes with breakdown voltages below 5.6 V enter breakdown via the Zener mechanism.

3.6.3 DIODE MODEL FOR THE BREAKDOWN REGION

In breakdown, the diode can be modeled by a voltage source of value V_Z in series with resistor R_Z, which sets the slope of the *i*-*v* characteristic in the breakdown region, as indicated in Fig. 3.14. The value of R_Z is normally small ($R_Z \leq 100\ \Omega$), and the reverse current flowing in the diode must be limited by the external circuit or the diode will be destroyed.

From the *i*-*v* characteristic in Fig. 3.14 and the model in Fig. 3.16, we see that the voltage across the diode is almost constant, independent of current, in the reverse-breakdown region. Some diodes are actually designed to be operated in **reverse breakdown.** These diodes are called **Zener diodes**[1] and have the special circuit symbol given in Fig. 3.16(b). Links to data sheets for a series of Zener diode can be found on the MCD website.

3.7 *pn* JUNCTION CAPACITANCE

Forward- and reverse-biased diodes have a capacitance associated with the *pn* junction. This capacitance is important under dynamic signal conditions because it prevents the voltage across the diode from changing instantaneously. The capacitance is referred to as the *pn* junction capacitance or the **depletion-layer capacitance.**

3.7.1 REVERSE BIAS

Under reverse bias, w_d increases beyond its zero-bias value, as expressed by Eq. (3.18), and hence the amount of charge in the depletion region also increases. Since the charge in the diode is changing with voltage, a capacitance results. Using Eqs. (3.4) and (3.7), the total space charge on the *n*-side

[1] The term *Zener diode* is typically used to refer to diodes that breakdown by either the Zener or avalanche mechanism.

of the diode is given by

$$Q_n = qN_D x_n A = q\left(\frac{N_A N_D}{N_A + N_D}\right) w_d A \qquad \text{C} \tag{3.20}$$

where A is the cross-sectional area of the diode and w_d is described by Eq. (3.18). The capacitance of the reverse-biased *pn* junction is given by

$$C_j = \frac{dQ_n}{dv_R} = \frac{C_{jo}A}{\sqrt{1 + \dfrac{v_R}{\phi_j}}} \qquad \text{where } C_{jo} = \frac{\varepsilon_s}{w_{do}} \qquad \text{F/cm}^2 \tag{3.21}$$

in which C_{jo} represents the **zero-bias junction capacitance** per unit area of the diode.

Equation (3.21) shows that the capacitance of the diode changes with applied voltage. The capacitance decreases as the reverse bias increases, exhibiting an inverse square root relationship. This voltage-controlled capacitance can be very useful in certain electronic circuits. Diodes can be designed with impurity profiles (called *hyper-abrupt profiles*) specifically optimized for operation as voltage-controlled capacitors. As for the case of Zener diodes, a special symbol exists for the variable capacitance diode, as shown in Fig. 3.17. Remember that this diode is designed to be operated under reverse bias, but it conducts in the forward direction. Links to data sheets for a series of variable capacitance diodes can be found on the MCD website.

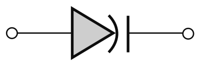


Figure 3.17 Circuit symbol for the variable capacitance diode (varactor).

EXERCISE: What is the value of C_{jo} for the diode in Ex. 3.1? What is the zero bias value of C_j if the diode junction area is 100 μm × 125 μm? What is the capacitance at a reverse bias of 5 V?

ANSWERS: 91.7 nf/cm^2; 11.5 pF; 4.64 pF

3.7.2 FORWARD BIAS

When the diode is operating under forward bias, additional charge is stored in the neutral regions near the edges of the space charge region. The amount of charge Q_D stored in the diode is proportional to the diode current:

$$Q_D = i_D \tau_T \qquad \text{C} \tag{3.22}$$

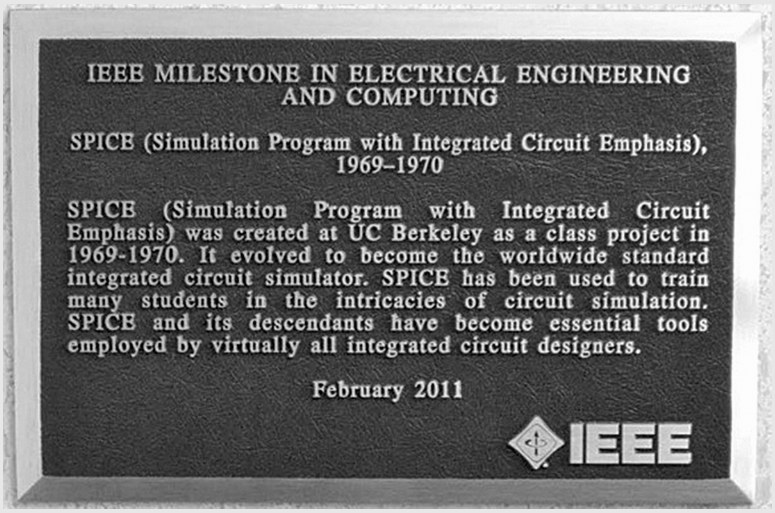
The proportionality constant τ_T is called the diode **transit time** and ranges from 10^{-15} s to more than 10^{-6} s (1 fs to 1 μs) depending on the size and type of diode. Because we know that i_D is dependent on the diode voltage through the diode equation, there is an additional capacitance, the **diffusion capacitance C_D,** associated with the forward region of operation:

$$C_D = \frac{dQ_D}{dv_D} = \frac{(i_D + I_S)\tau_T}{V_T} \cong \frac{i_D \tau_T}{V_T} \qquad \text{F} \tag{3.23}$$

in which V_T is the thermal voltage. The diffusion capacitance is proportional to current and can become quite large at high currents.

ELECTRONICS IN ACTION

The SPICE Circuit Simulation Program An IEEE Global History Network Milestone[2]



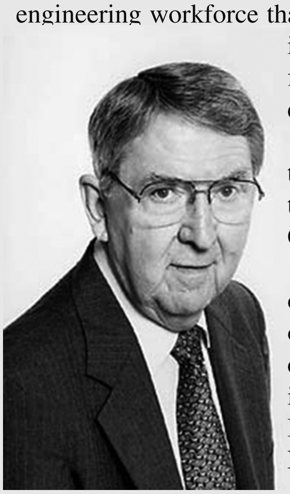
Courtesy of David Hodges

"SPICE (Simulation Program with Integrated Circuit Emphasis) was created at UC Berkeley as a class project in 1969–1970. It evolved to become the worldwide standard integrated circuit simulator. SPICE has been used to train many students in the intricacies of circuit simulation. SPICE and its descendants have become essential tools employed by virtually all integrated circuit designers.

SPICE was the first computer program for simulating the performance of integrated circuits that was readily available to undergraduate students for study of integrated circuit design. Hundreds of graduates from UC Berkeley and other universities became the backbone of the engineering workforce that moved the United States to microelectronics industry leadership in the 1970s. Graduates of Berkeley became leaders of today's largest firms delivering design automation capabilities for advanced microelectronics."

Above is a photograph of the SPICE Commemorative Plaque that may be found just inside the main entrance to Cory Hall, the Electrical Engineering Building at the University of California, Berkeley, CA. Cory Hall is the building where SPICE was developed.

Professor Donald O. Pederson guided the students that developed the SPICE program, and he was awarded the 1998 IEEE Medal of Honor "For creation of the SPICE program universally used for the computer aided design of circuits." Further information can be found in the *IEEE Solid-State Circuits Magazine*, "SPICE Commemorative Issue," vol. 3, no. 2, Spring 2011, and on the IEEE Global History Network website.



Professor Don Pederson, "Father of SPICE", IEEE Medal of Honor Recipient: *Courtesy of David Hodges*

[2] Quoted from the SPICE Circuit Simulation Program Milestone of the IEEE Global History Network: http://www.ieeeghn.org/wiki/index.php/Milestones:SPICE_Circuit_Simulation_Program Milestone photograph courtesy of Professor David A. Hodges, used with permission.

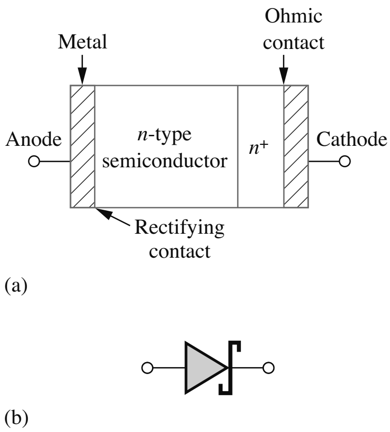


Figure 3.18 (a) Schottky barrier diode structure; (b) Schottky diode symbol.

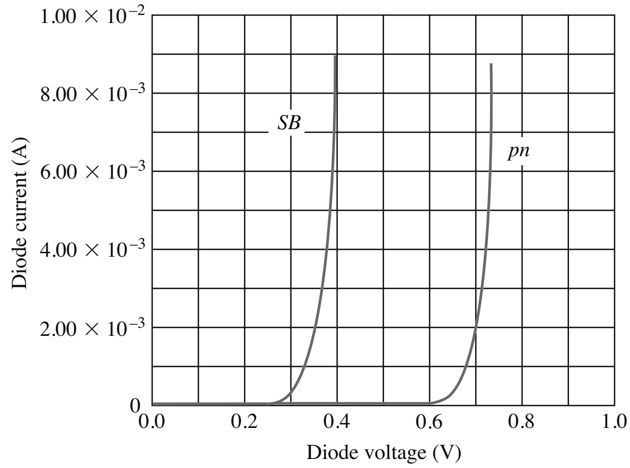


Figure 3.19 Comparison of *pn* junction (*pn*) and Schottky barrier diode (*SB*) *i*-*v* characteristics.

EXERCISE: A diode has a transit time of 10 ns. What is the diffusion capacitance of the diode for currents of 10 μA, 0.8 mA, and 50 mA at room temperature?

ANSWERS: 4 pF; 320 pF; 20 nF

3.8 SCHOTTKY BARRIER DIODE

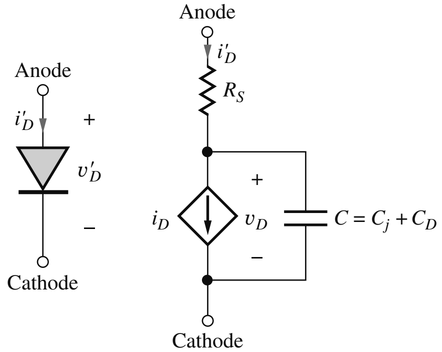
In a p^+n junction diode, the p-side is a highly doped region (a conductor), and one might wonder if it could be replaced with a metallic layer. That is in fact the case, and in the **Schottky barrier diode,** one of the semiconductor regions of the pn junction diode is replaced by a non-ohmic rectifying metal contact, as indicated in Fig. 3.18. It is easiest to form a Schottky contact to n-type silicon, and for this case the metal region becomes the diode anode. An n^+ region is added to ensure that the cathode contact is ohmic. The symbol for the Schottky barrier diode appears in Fig. 3.18(b).

The Schottky diode turns on at a much lower voltage than its pn-junction counterpart, as indicated in Fig. 3.19. It also has significantly reduced internal charge storage under forward bias. We encounter an important use of the Schottky diode in bipolar logic circuits in Chapter 9. Schottky diodes also find important applications in high-power rectifier circuits and fast switching applications.

3.9 DIODE SPICE MODEL AND LAYOUT

The circuit in Fig. 3.20 represents the diode model that is included in SPICE programs. Resistance R_S represents the inevitable series resistance that always accompanies fabrication of, and making contacts to, a real device structure. The current source represents the ideal exponential behavior of the diode as described by Eq. 3.12 and **SPICE parameters IS, N,** and V_T. The model equation for i_D also includes a second term, not shown here, that models the effects of carrier generation in the space charge region in a manner similar to Eq. (3.19).

The capacitor specification includes the depletion-layer capacitance for the reverse-bias region modeled by **SPICE parameters CJO, VJ, and M,** as well as the diffusion capacitance associated



$$i_D = \text{IS}\left[\exp\left(\frac{v_D}{NV_T}\right) - 1\right]$$

$$C_D = \text{TT}\,\frac{i_D}{NV_T} \quad \text{for} \quad v_D \geq 0$$

$$C_j = \frac{\text{CJO}}{\left(1 - \frac{v_D}{\text{VJ}}\right)^M}\ \text{RAREA} \quad \text{for} \quad v_D \leq 0$$

Figure 3.20 Diode equivalent circuit and simplified versions of the model equations used in SPICE programs.

with the junction under forward bias and defined by N and the transit-time parameter TT. In SPICE, the "junction grading coefficient" is an adjustable parameter. Using the typical value of $M = 0.5$ results in Eq. (3.21).

EXERCISE: Find the default values of the seven parameters in Table 3.1 for the SPICE program that you use in class. Compare to the values in Table 3.1.

TABLE 3.1
SPICE Diode Parameter Equivalences

PARAMETER	OUR TEXT	SPICE	TYPICAL DEFAULT VALUES
Saturation current	I_S	IS	10 fA
Ohmic series resistance	R_S	RS	0 Ω
Ideality factor or emission coefficient	n	N	1
Transit time	τ_T	TT	0 s
Zero-bias junction capacitance for a unit area diode RAREA = 1	$C_{jo} \cdot A$	CJO	0 F
Built-in potential	ϕ_j	VJ	1 V
Junction grading coefficient	—	M	0.5
Relative junction area	—	RAREA	1

3.9.1 DIODE LAYOUT

Figure 3.21(a) shows the layout of a simple diode fabricated by forming a p-type diffusion in an n-type silicon wafer, as outlined in Chapter 2. This diode has a long rectangular p-type diffusion to increase the value of I_S, which is proportional to the junction area. Multiple contacts are formed to the p-type anode, and the p-region is surrounded by a collar of contacts to the n-type region. Both these sets of contacts are used to minimize the value of the extrinsic series resistance R_S of the diode, as included in the model in Fig. 3.20. Identical contacts are used so that they all tend to etch open at the same time during the fabrication process. The use of multiple identical contacts also facilitates calculation of the overall contact resistance. Heavily doped n-type regions are placed under the n-region contacts to ensure formation of an ohmic contact and prevent formation of a Schottky barrier diode.

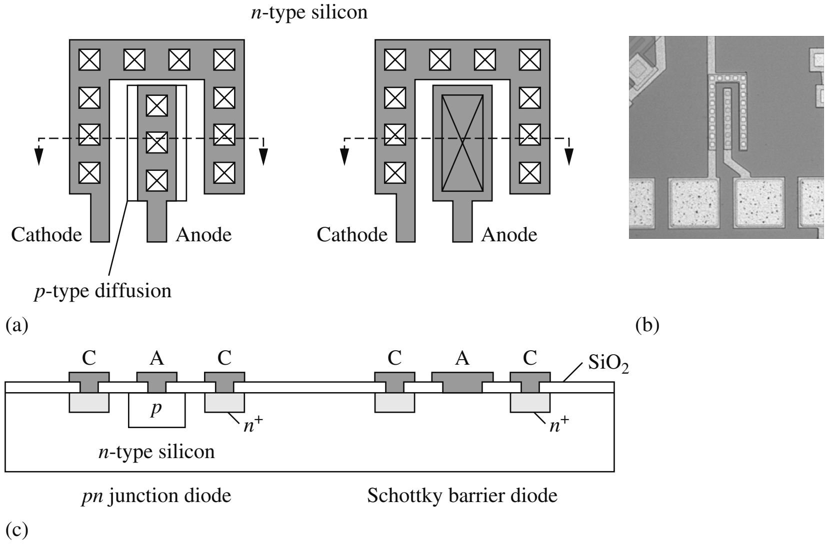


Figure 3.21 (a) Layout of a *pn* junction diode and a Schottky diode (b) *pn* junction diode photograph (c) Cross-section of the two diodes (See top view of diode in Chapter 3 opener.)

A conceptual drawing of a metal-semiconductor or Schottky diode also appears in Fig. 3.21(a) in which the aluminum metallization acts as the anode of the diode and the n-type semiconductor is the diode cathode. Careful attention to processing details is needed to form a diode rather than just an ohmic contact.

3.10 DIODE CIRCUIT ANALYSIS

We now begin our analysis of circuits containing diodes and introduce simplified circuit models for the diode. Figure 3.22 presents a series circuit containing a voltage source, resistor, and diode. Note that V and R may represent the Thévenin equivalent of a more complicated two-terminal network. Also note the notational change in Fig. 3.22. In the circuits that we analyze in the next few sections, the applied voltage and resulting diode voltage and current will all be dc quantities. (Recall that the dc components of the total quantities i_D and v_D are indicated by I_D and V_D, respectively.)

One common objective of diode circuit analysis is to find the **quiescent operating point** (Q-point), or **bias point,** for the diode. The Q-point consists of the dc current and voltage (I_D, V_D) that define the point of operation on the diode's i-v characteristic. We start the analysis by writing the loop equation for the circuit of Fig. 3.22:

$$V = I_D R + V_D \tag{3.24}$$

Equation (3.24) represents a constraint placed on the diode operating point by the circuit elements. The diode i-v characteristic in Fig. 3.8 represents the allowed values of I_D and V_D as determined by the solid-state diode itself. Simultaneous solution of these two sets of constraints defines the Q-point.

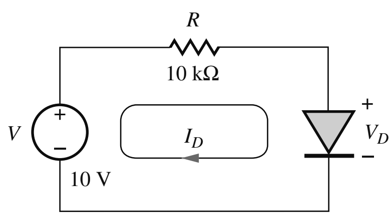


Figure 3.22 Diode circuit containing a voltage source and resistor.

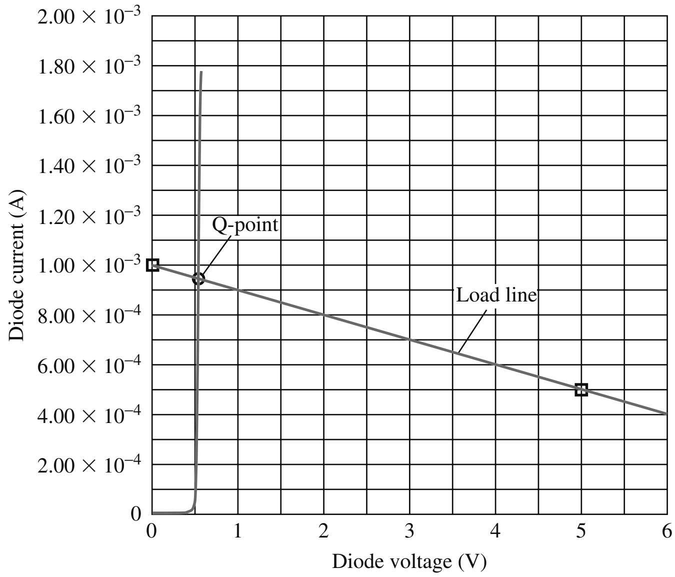


Figure 3.23 Diode *i*-*v* characteristic and load line.

We explore several methods for determining the solution to Eq. (3.24), including graphical analysis and the use of models of varying complexity for the diode. These techniques will include

- Graphical analysis using the load-line technique.
- Analysis with the mathematical model for the diode.
- Simplified analysis with an ideal diode model.
- Simplified analysis using the constant voltage drop model.

3.10.1 LOAD-LINE ANALYSIS

In some cases, the *i*-*v* characteristic of the solid-state device may be available only in graphical form, as in Fig. 3.23. We can then use a graphical approach (**load-line analysis**) to find the simultaneous solution of Eq. (3.24) with the graphical characteristic. Equation (3.24) defines the **load line** for the diode. The Q-point can be found by plotting the graph of the load line on the i-v characteristic for the diode. The intersection of the two curves represents the quiescent operating point, or Q-point, for the diode.

EXAMPLE 3.5 **LOAD-LINE ANALYSIS**

The graphical load-line approach is an important concept for visualizing the behavior of diode circuits as well as for estimating the actual Q-point.

PROBLEM Use load-line analysis to find the Q-point for the diode circuit in Fig. 3.22 using the i-v characteristic in Fig. 3.23.

SOLUTION **Known Information and Given Data:** The diode i-v characteristic is presented graphically in Fig. 3.23. Diode circuit is given in Fig. 3.22 with $V = 10$ V and $R = 10$ kΩ.

Unknowns: Diode Q-point (I_D, V_D).

Approach: Write the load-line equation and find two points on the load line that can be plotted on the graph in Fig. 3.23. The Q-point is at the intersection of the load line with the diode i-v characteristic.

Assumptions: Diode temperature corresponds to the temperature at which the graph in Fig. 3.23 was measured.

Analysis: Using the values from Fig. 3.22, Eq. (3.24) can be rewritten as

$$10 = 10^4 I_D + V_D \tag{3.25}$$

Two points are needed to define the line. The simplest choices are

$$I_D = (10\ \text{V}/10\ \text{k}\Omega) = 1\ \text{mA} \quad \text{for} \quad V_D = 0 \qquad \text{and} \qquad V_D = 10\ \text{V} \quad \text{for} \quad I_D = 0$$

Unfortunately, the second point is not in the range of the graph presented in Fig. 3.23, but we are free to choose any point that satisfies Eq. (3.25). Let's pick $V_D = 5$ V:

$$I_D = (10 - 5)\text{V}/10^4\ \Omega = 0.5\ \text{mA} \qquad \text{for } V_D = 5$$

These points and the resulting load line are plotted in Fig. 3.23. The Q-point is given by the intersection of the load line and the diode characteristic:

$$\text{Q-point} = (0.95\ \text{mA}, 0.6\ \text{V})$$

Check of Results: We can double check our result by substituting the diode voltage found from the graph into Eq. (3.25) and calculating I_D. Using $V_D = 0.6$ V in Eq. (3.25) yields an improved estimate for the Q-point: (0.94 mA, 0.6 V). [We could also substitute 0.95 mA into Eq. (3.25) and calculate V_D.]

Discussion: Note that the values determined graphically are not quite on the load line since they do not precisely satisfy the load-line equation. This is a result of the limited precision that we can obtain by reading the graph.

EXERCISE: Repeat the load-line analysis if $V = 5$ V and $R = 5$ kΩ.

ANSWERS: (0.88 mA, 0.6 V)

EXERCISE: Use SPICE to find the Q-point for the circuit in Fig. 3.22. Use the default values of parameters in your SPICE program.

ANSWERS: (935 μA, 0.653 V) for $I_S = 10$ fA and $T = 300$ K

3.10.2 ANALYSIS USING THE MATHEMATICAL MODEL FOR THE DIODE

We can use our mathematical model for the diode to approach the solution of Eq. (3.25) more directly. The particular diode characteristic in Fig. 3.23 is represented quite accurately by diode Eq. (3.11), with $I_S = 10^{-13}$ A, $n = 1$, and $V_T = 0.025$ V:

$$I_D = I_S \left[\exp\left(\frac{V_D}{V_T}\right) - 1\right] = 10^{-13}[\exp(40V_D) - 1] \tag{3.26}$$

Eliminating I_D by substituting Eq. (3.26) into Eq. (3.25) yields

$$10 = 10^4 \cdot 10^{-13}[\exp(40V_D) - 1] + V_D \tag{3.27}$$

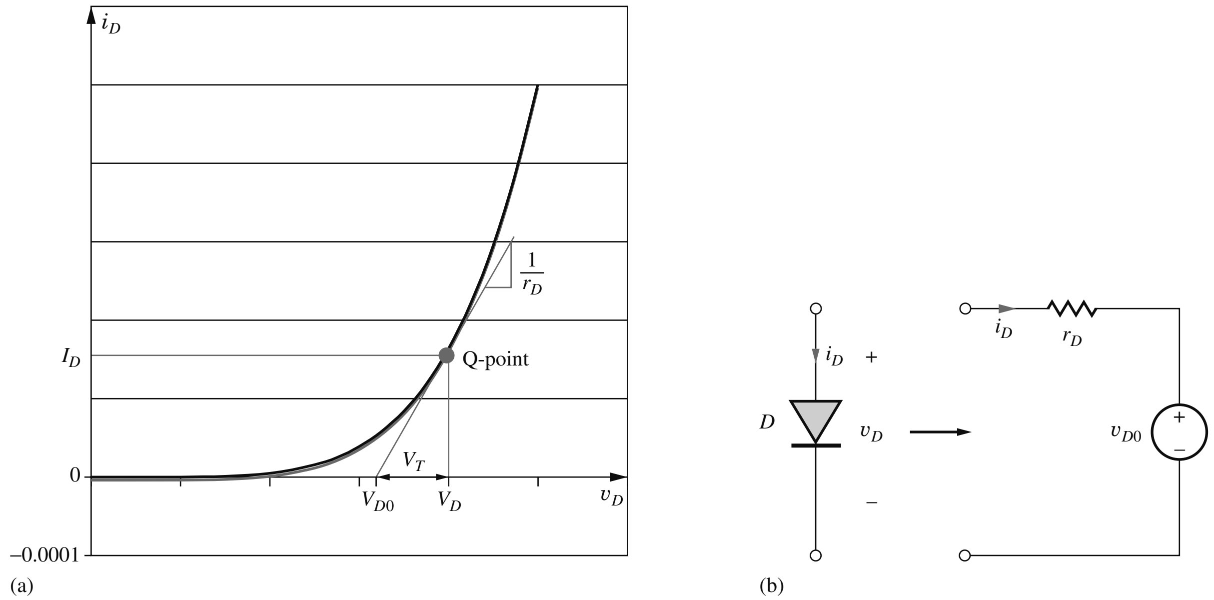


Figure 3.24 (a) Diode behavior around the Q-point; (b) linear model for the diode at the Q-point.

The expression in Eq. (3.27) is called a *transcendental equation* and does not have a closed-form analytical solution, so we settle for a numerical answer to the problem.

One approach to finding a numerical solution to Eq. (3.27) is through simple trial and error. We can guess a value of V_D and see if it satisfies Eq. (3.27). Based on the result, a new guess can be formulated and Eq. (3.27) evaluated again. The human brain is quite good at finding a sequence of values that will converge to the desired solution.

On the other hand, it is often preferable to use a computer to find the solution to Eq. (3.27), particularly if we need to find the answer to several different problems or parameter sets. The computer, however, requires a much more well-defined iteration strategy than brute force trial and error.

We can develop an iterative solution method for the diode circuit in Fig. 3.22 by creating a linear model for the diode equation in the vicinity of the diode Q-point as depicted in Fig. 3.24(a). First we find the slope of the diode characteristic at the operating point:

$$g_D = \left.\frac{\partial i_D}{\partial v_D}\right|_{Q-Pt} = \frac{I_S}{V_T}\exp\left(\frac{V_D}{V_T}\right) = \frac{I_D + I_S}{V_T} \cong \frac{I_D}{V_T} \quad \text{and} \quad r_D = \frac{1}{g_D} = \frac{V_T}{I_D} \tag{3.28}$$

Slope g_D is called the diode conductance, and its reciprocal r_D is termed the diode resistance. Next we can use the slope to find the x-axis intercept point V_{D0}:

$$V_{D0} = V_D - I_D r_D = V_D - V_T \tag{3.29}$$

V_{D0} and r_D represent a two-element linear circuit model for the diode as in Fig. 3.24(b), and this circuit model replaces the diode in the single loop circuit in Fig. 3.25.

Now we can use an iterative process to find the Q-point of the diode in the circuit.

1. Pick a starting guess for I_D.
2. Calculate the diode voltage using $V_D = V_T \ln\left(1 + \frac{I_D}{I_S}\right)$.
3. Calculate the values of V_{D0} and r_D.
4. Calculate a new estimate for I_D from the circuit in Fig. 3.25(b): $I_D = \frac{V - V_{D0}}{R + r_D}$.
5. Repeat steps 2 to 4 until convergence is obtained.

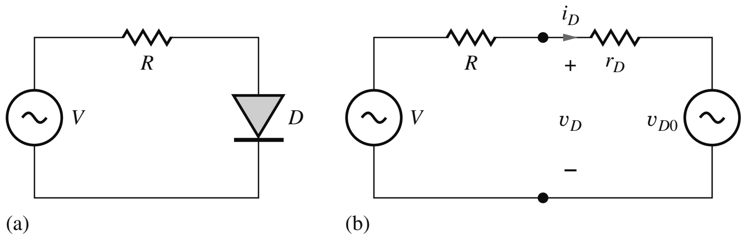


Figure 3.25 (a) Diode circuit; (b) circuit with two-element diode model.

TABLE 3.2
Example of Iterative Analysis

I_D (A)	V_D (V)	R_D (Ω)	V_{D0} (V)
1.0000E-03	0.5756	25.80	0.5498
9.4258E-04	0.5742	27.37	0.5484
9.4258E-04	0.5742	27.37	0.5484

Table 3.2 presents the results of performing the above iteration process using a spreadsheet. The diode current and voltage converge rapidly in only three iterations.

Note that one can achieve answers to an almost arbitrary precision using the numerical approach. However, in most real circuit situations, we will not have an accurate value for the saturation current of the diode, and there will be significant tolerances associated with the sources and passive components in the circuit. For example, the saturation current specification for a given diode type may vary by factors ranging from 10:1 to as much as 100:1. In addition, resistors commonly have ± 5 percent or ± 10 percent tolerances, and we do not know the exact operating temperature of the diode (remember the -1.8 mV/K temperature coefficient) or the precise value of the parameter n. Hence, it does not make sense to try to obtain answers with a precision of more than a few significant digits.

An alternative to the use of a spreadsheet is to write a simple program using a high-level language. The solution to Eq. (3.28) also can be found using the "solver" routines in many calculators, which use iteration procedures more sophisticated than that just described. MATLAB also provides the function fzero, which will calculate the zeros of a function as outlined in Ex. 3.6.

EXERCISE: An alternative expression (another transcendental equation) for the basic diode circuit can be found by eliminating V_D in Eq. (3.25) using Eq. (3.14). Show that the result is

$$10 = 10^4 I_D + 0.025 \ln\left(1 + \frac{I_D}{I_S}\right)$$

EXAMPLE 3.6 SOLUTION OF THE DIODE EQUATION USING MATLAB

MATLAB is one example of a computer tool that can be used to find the solution to transcendental equations.

PROBLEM Use MATLAB to find the solution to Eq. (3.27).

SOLUTION **Known Information and Given Data:** Diode circuit in Fig. 3.22 with $V = 10$ V, $R = 10$ kΩ, $I_S = 10^{-13}$ A, $n = 1$, and $V_T = 0.025$ V

Unknowns: Diode voltage V_D

Approach: Create a MATLAB "M-File" describing Eq. (3.27). Execute the program to find the diode voltage.

Assumptions: Room temperature operation with $V_T = 1/40$ V

Analysis: First, create an M-File for the function 'diode':

```
function xd = diode(vd)
xd = 10 - (10^(-9)) * (exp(40 * vd) - 1) - vd;
```

Then find the solution near 1 V:

```
fzero('diode',1)
```

Answer: 0.5742 V

Check of Results: The diode voltage is positive and in the range of 0.5 to 0.8 V, which is expected for a diode. Substituting this value of voltage into the diode equation yields a current of 0.944 mA. This answer appears reasonable since we know that the diode current cannot exceed 10 V/10 kΩ = 1.0 mA, which is the maximum current available from the circuit [i.e., if the diode were replaced with a short circuit ($V_D = 0$), the current in the circuit would be 1 mA]. See Sec. 3.10.3.

EXERCISE: Use the MATLAB to find the solution to

$$10 = 10^4 I_D + 0.025 \ \ln\left(1 + \frac{I_D}{I_S}\right) \qquad \text{for } I_S = 10^{-13}\ \text{A}$$

ANSWER: 942.6 μA

EXAMPLE 3.7 EFFECT OF DEVICE TOLERANCES ON DIODE Q-POINTS

Let us now see how sensitive our Q-point results are to the exact value of the diode saturation current.

PROBLEM Suppose that there is a tolerance on the value of the saturation current such that the value is given by

$$I_S^{\text{nom}} = 10^{-15}\ \text{A} \qquad \text{and} \qquad 2 \times 10^{-16}\ \text{A} \le I_S \le 5 \times 10^{-15}\ \text{A}$$

Find the nominal, smallest, and largest values of the diode voltage and current in the circuit in Fig. 3.22.

SOLUTION **Known Information and Given Data:** The nominal and worst-case values of saturation current are given as well as the circuit values in Fig. 3.22.

Unknowns: Nominal and worst-case values for the diode Q-point: (I_D, V_D)

Approach: Use MATLAB or the solver on our calculator to find the diode voltages and then the currents for the nominal and worst-case values of I_S. Note from Eq. (3.24) that the maximum value of diode voltage corresponds to minimum current and vice versa.

Assumptions: Room temperature operation with $V_T = 0.025$ V. The voltage and resistance in the circuit do not have tolerances associated with them.

Analysis: For the nominal case, Eq. (3.28) becomes

$$f = 10 - 10^4(10^{-15})[\exp(40V_D) - 1] - V_D$$

for which the solver yields

$$V_D^{\text{nom}} = 0.689 \text{ V} \qquad \text{and} \qquad I_D^{\text{nom}} = \frac{(10 - 0.689) \text{ V}}{10^4 \ \Omega} = 0.931 \text{ mA}$$

For the minimum I_S case, Eq. (3.28) is

$$f = 10 - 10^4(2 \times 10^{-16})[\exp(40V_D) - 1] - V_D$$

and the solver yields

$$V_D^{\max} = 0.729 \text{ V} \qquad \text{and} \qquad I_D^{\min} = \frac{(10 - 0.729) \text{ V}}{10^4 \ \Omega} = 0.927 \text{ mA}$$

Finally, for the maximum value of I_S, Eq. (3.28) becomes

$$f = 10 - 10^4(5 \times 10^{-15})[\exp(40V_D) - 1] - V_D$$

and the solver gives

$$V_D^{\min} = 0.649 \text{ V} \qquad \text{and} \qquad I_D^{\max} = \frac{(10 - 0.649) \text{ V}}{10^4 \ \Omega} = 0.935 \text{ mA}$$

Check of Results: The diode voltages are positive and in the range of 0.5 to 0.8 V which is expected for a diode. The diode currents are all less than the short circuit current available from the voltage source ($10 \text{ V}/10 \text{ k}\Omega = 1.0 \text{ mA}$).

Discussion: Note that even though the diode saturation current in this circuit changes by a factor of 5:1 in either direction, the current changes by less than ±0.5%. As long as the driving voltage in the circuit is much larger than the diode voltage, the current should be relatively insensitive to changes in the diode voltage or the diode saturation current.

EXERCISE: Find V_D and I_D if the upper limit on I_S is increased to 10^{-14} A.

ANSWERS: 0.6316 V; 0.9368 A

EXERCISE: Use the Solver function in your calculator to find the solution to

$$10 = 10^4 I_D + 0.025 \ \ln\left(1 + \frac{I_D}{I_S}\right) \quad \text{for } I_S = 10^{-13} \text{ A} \qquad \text{and} \qquad I_S = 10^{-15} \text{ A}$$

ANSWERS: 0.9426 mA; 0.9311 mA

3.10.3 THE IDEAL DIODE MODEL

Graphical load-line analysis provides insight into the operation of the diode circuit of Fig. 3.22, and the mathematical model can be used to provide more accurate solutions to the load-line problem. The next method discussed provides simplified solutions to the diode circuit of Fig. 3.22 by introducing simplified diode circuit models of varying complexity.

The diode, as described by its *i-v* characteristic in Fig. 3.8 or by Eq. (3.11), is obviously a nonlinear device. However, most, if not all, of the circuit analysis that we have learned in electrical engineering thus far assumed that the circuits were composed of linear elements. To use this wealth of analysis techniques, we will use **piecewise linear** approximations to the diode characteristic.

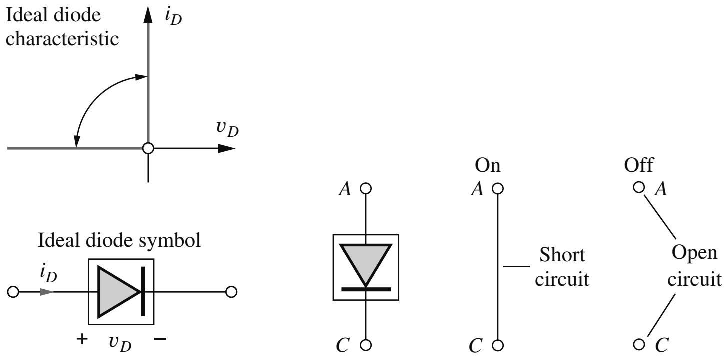


Figure 3.26 (a) Ideal diode i-v characteristics and circuit symbol; (b) circuit models for on and off states of the ideal diode.

The **ideal diode model** is the simplest model for the diode. The i-v characteristic for the **ideal diode** in Fig. 3.26 consists of two straight-line segments. If the diode is conducting a forward or positive current (forward-biased), then the voltage across the diode is zero. If the diode is reverse-biased, with $v_D < 0$, then the current through the diode is zero. These conditions can be stated mathematically as

$$v_D = 0 \quad \text{for } i_D > 0 \qquad \text{and} \qquad i_D = 0 \quad \text{for } v_D \le 0$$

The special symbol in Fig. 3.26 is used to represent the ideal diode in circuit diagrams.

We can now think of the diode as having two states. The diode is either conducting in the *on* state, or nonconducting and *off*. For circuit analysis, we use the models in Fig. 3.26(b) for the two states. If the diode is on, then it is modeled by a "short" circuit, a wire. For the off state, the diode is modeled by an "open" circuit, no connection.

Analysis Using the Ideal Diode Model

Let us now analyze the circuit of Fig. 3.22 assuming that the diode can be modeled by the ideal diode of Fig. 3.26(b). The diode has two possible states, and our analysis of diode circuits proceeds as follows:

1. Select a model for the diode.
2. Identify the anode and cathode of the diode and label the diode voltage v_D and current i_D.
3. Make an (educated) guess concerning the region of operation of the diode based on the circuit configuration.
4. Analyze the circuit using the diode model appropriate for the assumption in step 3.
5. Check the results to see if they are consistent with the assumptions.

For this analysis, we select the ideal diode model. The diode in the original circuit is replaced by the ideal diode, as in Fig. 3.27(b). Next we must guess the state of the diode. Because the voltage source appears to be trying to force a positive current through the diode, our first guess will be to

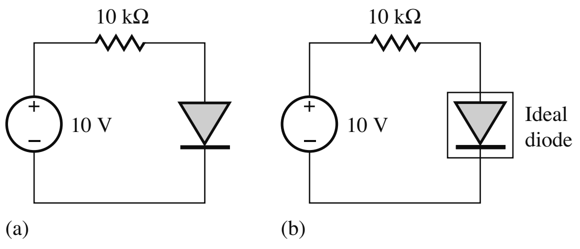


Figure 3.27 (a) Original diode circuit; (b) circuit modeled by an ideal diode.

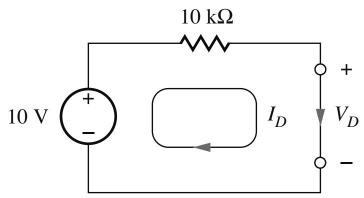


Figure 3.28 Ideal diode replaced with its model for the on state.

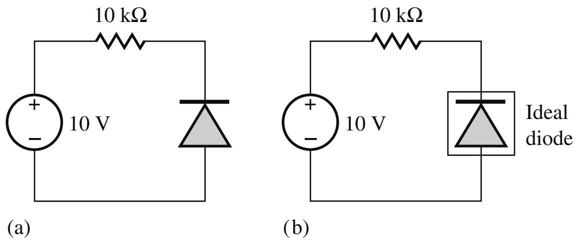


Figure 3.29 (a) Circuit with reverse-biased diode; (b) circuit modeled by ideal diode.

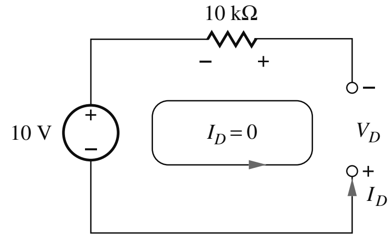


Figure 3.30 Ideal diode replaced with its model for the off region.

assume that the diode is on. The ideal diode of Fig. 3.27(b) is replaced by its piecewise linear model for the on region in Fig. 3.28, and the diode current is given by

$$I_D = \frac{(10 - 0)\text{ V}}{10\text{ k}\Omega} = 1.00\text{ mA}$$

The current $I_D \geq 0$, which is consistent with the assumption that the diode is on. Based on the ideal diode model, we find that the diode is forward-biased and operating with a current of 1 mA. The Q-point is therefore equal to (1 mA, 0 V).

Analysis of a Circuit Containing a Reverse-Biased Diode

A second circuit example in which the diode terminals have been reversed appears in Fig. 3.29; the ideal diode model is again used to model the diode [Fig. 3.29(b)]. The voltage source now appears to be trying to force a current backward through the diode. Because the diode cannot conduct in this direction, we assume the diode is off. The ideal diode of Fig. 3.29(b) is replaced by the open circuit model for the off region, as in Fig. 3.30.

Writing the loop equation for this case,

$$10 + V_D + 10^4 I_D = 0$$

Because $I_D = 0$, $V_D = -10$ V. The calculated diode voltage is negative, which is consistent with the starting assumption that the diode is off. The analysis shows that the diode in the circuit of Fig. 3.29 is indeed reverse-biased. The Q-point is (0, −10 V).

Although these two problems may seem rather simple, the complexity of diode circuit analysis increases rapidly as the number of diodes increases. If the circuit has N diodes, then the number of possible states is 2^N. A circuit with 10 diodes has 1024 different possible circuits that could be analyzed! Only through practice can we develop the intuition needed to avoid analysis of many incorrect cases. We analyze more complex circuits shortly, but first let's look at a slightly better piecewise linear model for the diode.

3.10.4 CONSTANT VOLTAGE DROP MODEL

We know from our earlier discussion that there is a small, nearly constant voltage across the forward-biased diode. The ideal diode model ignores the presence of this voltage. However, the piecewise linear model for the diode can be improved by adding a constant voltage V_{on} in series with the ideal diode, as shown in Fig. 3.31(b). This is the **constant voltage drop (CVD) model.** V_{on} offsets the *i-v* characteristic of the ideal diode, as indicated in Fig. 3.31(c). The piecewise linear models for the two states become a voltage source V_{on} for the on state and an open circuit for the off state. We now have

$$v_D = V_{on} \quad \text{for } i_D > 0 \qquad \text{and} \qquad i_D = 0 \quad \text{for } v_D \leq V_{on}$$

We may consider the ideal diode model to be the special case of the constant voltage drop model for which $V_{on} = 0$. From the *i-v* characteristics presented in Fig. 3.8, we see that a reasonable choice for V_{on} is 0.6 to 0.7 V. We use a voltage of 0.6 V as the turn-on voltage for our diode circuit analysis.

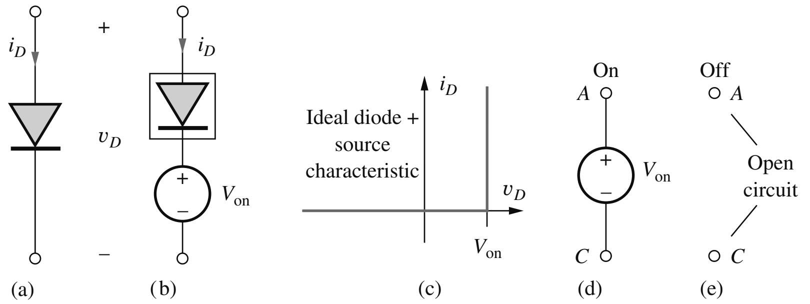


Figure 3.31 Constant voltage drop model for diode: (a) actual diode; (b) ideal diode plus voltage source V_{on}; (c) composite i-v characteristic; (d) CVD model for the on state; (e) model for the off state.

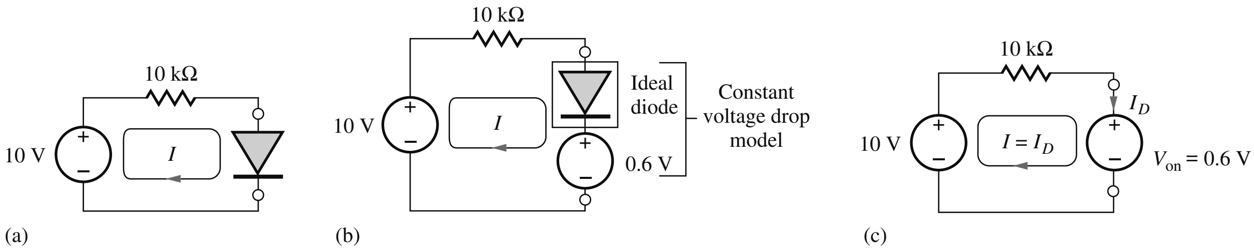


Figure 3.32 Diode circuit analysis using constant voltage drop model: (a) original diode circuit; (b) circuit with diode replaced by the constant voltage drop model; (c) circuit with ideal diode replaced by the piecewise linear model.

Diode Analysis with the Constant Voltage Drop Model

Let us analyze the diode circuit from Fig. 3.22 using the CVD model for the diode. The diode in Fig. 3.32(a) is replaced by its CVD model in Fig. 3.32(b). The 10-V source once again appears to be forward biasing the diode, so assume that the ideal diode is on, resulting in the simplified circuit in Fig. 3.32(c). The diode current is given by

$$I_D = \frac{(10 - V_{on})\text{ V}}{10\text{ k}\Omega} = \frac{(10 - 0.6)\text{ V}}{10\text{ k}\Omega} = 0.940\text{ mA} \tag{3.30}$$

which is slightly smaller than that predicted by the ideal diode model but quite close to the exact result found earlier in Ex. 3.6.

3.10.5 MODEL COMPARISON AND DISCUSSION

We have analyzed the circuit of Fig. 3.22 using four different approaches; the various results appear in Table 3.3. All four sets of predicted voltages and currents are quite similar. Even the simple ideal diode model only overestimates the current by less than 10 percent compared to the mathematical model. We see that the current is quite insensitive to the actual choice of diode voltage. This is a result of the exponential dependence of the diode current on voltage as well as the large source voltage (10 V) in this particular circuit.

TABLE 3.3
Comparison of Diode Circuit Analysis Results

ANALYSIS TECHNIQUE	DIODE CURRENT	DIODE VOLTAGE
Load-line analysis	0.94 mA	0.6 V
Mathematical model	0.942 mA	0.547 V
Ideal diode model	1.00 mA	0 V
Constant voltage drop model	0.940 mA	0.600 V

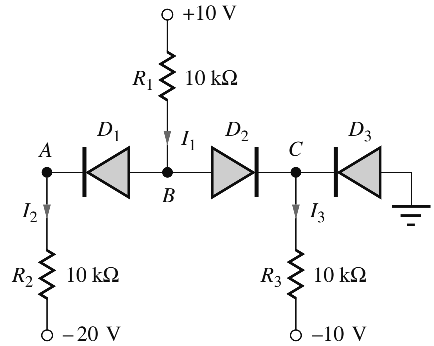


Figure 3.33 Example of a circuit containing three diodes.

TABLE 3.4
Possible Diode States for Circuit in Fig. 3.33

D_1	D_2	D_3
Off	Off	Off
Off	Off	On
Off	On	Off
Off	On	On
On	Off	Off
On	Off	On
On	On	Off
On	On	On

Rewriting Eq. (3.30),

$$I_D = \frac{10 - V_{on}}{10\ \text{k}\Omega} = \frac{10\ \text{V}}{10\ \text{k}\Omega}\left(1 - \frac{V_{on}}{10}\right) = (1.00\ \text{mA})\left(1 - \frac{V_{on}}{10}\right) \tag{3.31}$$

we see that the value of I_D is approximately 1 mA for $V_{on} \ll 10$ V. Variations in V_{on} have only a small effect on the result. However, the situation would be significantly different if the source voltage were only 1 V for example (see Prob. 3.62).

3.11 MULTIPLE-DIODE CIRCUITS

The load-line technique is applicable only to single-diode circuits, and the mathematical model, or numerical iteration technique, becomes much more complex for circuits with more than one nonlinear element. In fact, the SPICE electronic circuit simulation program referred to throughout this book is designed to provide numerical solutions to just such complex problems. However, we also need to be able to perform hand analysis to predict the operation of multidiode circuits as well as to build our understanding and intuition for diode circuit operation. In this section we discuss the use of the simplified diode models for hand analysis of more complicated diode circuits.

As the complexity of diode circuits increases, we must rely on our intuition to eliminate unreasonable solution choices. Even so, analysis of diode circuits may require several iterations. Intuition can only be developed over time by working problems, and here we analyze a circuit containing three diodes.

Figure 3.33 is an example of a circuit with several diodes. In the analysis of this circuit, we will use the CVD model to improve the accuracy of our hand calculations.

EXAMPLE 3.8 ANALYSIS OF A CIRCUIT CONTAINING THREE DIODES

Now we will attempt to find the solution for a three-diode circuit. Our analysis will employ the CVD model.

PROBLEM Find the Q-points for the three diodes in Fig. 3.33. Use the constant voltage drop model for the diodes.

SOLUTION **Known Information and Given Data:** Circuit topology and element values in Fig. 3.33

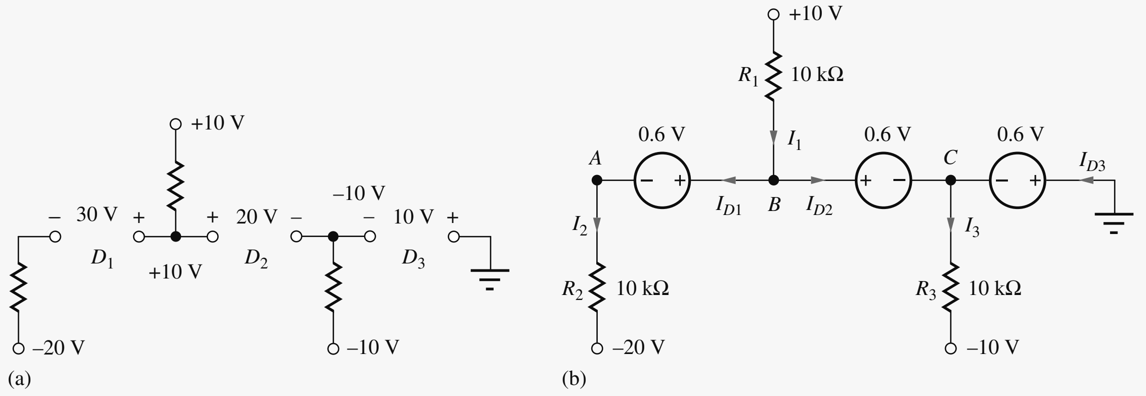


Figure 3.34 (a) Three diode circuit model with all diodes off; (b) circuit model for circuit of Fig. 3.33 with all diodes on.

Unknowns: (I_{D1}, V_{D1}), (I_{D2}, V_{D2}), (I_{D3}, V_{D3})

Approach: With three diodes, there are the eight On/Off combinations indicated in Table 3.4. A common method that we often use to find a starting point is to consider the circuit with all the diodes in the off state as in Fig 3.34(a). Here we see that the circuit tends to produce large forward biases across D_1, D_2, and D_3. So our second step will be to assume that all the diodes are on.

Assumptions: Use the constant voltage drop model with $V_{on} = 0.6$ V.

Analysis: The circuit is redrawn using the CVD diode models in Fig. 3.34(b). Here we skipped the step of physically drawing the circuit with the ideal diode symbols but instead incorporated the piecewise linear models directly into the figure. Working from right to left, we see that the voltages at nodes C, B, and A are given by

$$V_C = -0.6\,\text{V} \qquad V_B = -0.6 + 0.6 = 0\,\text{V} \qquad V_A = 0 - 0.6 = -0.6\,\text{V}$$

With the node voltages specified, it is easy to find the current through each resistor:

$$I_1 = \frac{10 - 0}{10}\,\frac{\text{V}}{\text{k}\Omega} = 1\,\text{mA} \qquad I_2 = \frac{-0.6 - (-20)}{10}\,\frac{\text{V}}{\text{k}\Omega} = 1.94\,\text{mA}$$

$$I_3 = \frac{-0.6 - (-10)}{10}\,\frac{\text{V}}{\text{k}\Omega} = 0.94\,\text{mA} \tag{3.32}$$

Using Kirchhoff's current law, we also have

$$I_2 = I_{D1} \qquad I_1 = I_{D1} + I_{D2} \qquad I_3 = I_{D2} + I_{D3} \tag{3.33}$$

Combining Eqs. (3.32) and (3.33) yields the three diode currents:

$$I_{D1} = 1.94\,\text{mA} > 0\,✔ \qquad I_{D2} = -0.94\,\text{mA} < 0\,\times \qquad I_{D3} = 1.86\,\text{mA} > 0\,✔ \tag{3.34}$$

Check of Results: I_{D1} and I_{D3} are greater than zero and therefore consistent with the original assumptions. However, I_{D2}, which is less than zero, represents a contradiction. So we must try again.

SECOND ITERATION For our second attempt, let us assume D_1 and D_3 are on and D_2 is off, as in Fig. 3.35(a). We now have

$$+10 - 10{,}000I_1 - 0.6 - 10{,}000I_2 + 20 = 0 \qquad \text{with } I_1 = I_{D1} = I_2 \tag{3.35}$$

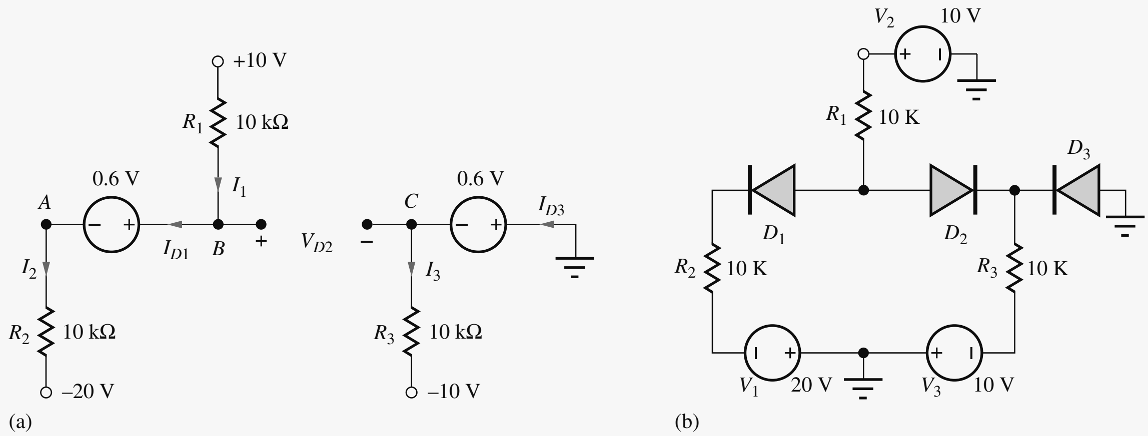


Figure 3.35 (a) Circuit with diodes D_1 and D_3 on and D_2 off; (b) Circuit for SPICE simulation.

which yields

$$I_{D1} = \frac{29.4}{20}\frac{\text{V}}{\text{k}\Omega} = 1.47\,\text{mA} > 0 \checkmark$$

Also

$$I_{D3} = I_3 = \frac{-0.6-(-10)}{10}\frac{\text{V}}{\text{k}\Omega} = 0.940\,\text{mA} > 0 \checkmark$$

The voltage across diode D_2 is given by

$$V_{D2} = 10 - 10{,}000I_1 - (-0.6) = 10 - 14.7 + 0.6 = -4.10\,\text{V} < 0 \checkmark$$

Check of Results: I_{D1}, I_{D3}, and V_{D2} are now all consistent with the circuit assumptions, so the Q-points for the circuit are

$$D_1\text{: }(1.47\,\text{mA}, 0.6\,\text{V}) \qquad D_2\text{: }(0\,\text{mA}, -4.10\,\text{V}) \qquad D_3\text{: }(0.940\,\text{mA}, 0.6\,\text{V})$$

Discussion: The Q-point values that we would have obtained using the ideal diode model are (see Prob. 3.73):

$$D_1\text{: }(1.50\,\text{mA}, 0\,\text{V}) \qquad D_2\text{: }(0\,\text{mA}, -5.00\,\text{V}) \qquad D_3\text{: }(1.00\,\text{mA}, 0\,\text{V})$$

The values of I_{D1} and I_{D3} differ by less than 6 percent. However, the reverse-bias voltage on D_2 differs by 20 percent. This shows the difference that the choice of models can make. The results from the circuit using the CVD model should be a more accurate estimate of how the circuit will actually perform than would result from the ideal diode case. Remember, however, that these calculations are both just approximations based on our models for the actual behavior of the real diode circuit.

Computer-Aided Analysis: SPICE analysis yields the following Q-points for the circuit in Fig. 3.35(b): (1.47 mA, 0.665 V), (−4.02 pA, −4.01 V), (0.935 mA, 0.653 V). Device parameter and Q-point information are found directly using the SHOW and SHOWMOD commands in SPICE. Or, voltmeters and ammeters (zero-valued current and voltage sources) can be inserted in the circuit in some implementations of SPICE. Note that the −4 pA current in D_2 is much larger than the reverse saturation current of the diode (IS defaults to 10 fA), and results from a more complete SPICE model in the author's version of SPICE.

EXERCISE: Find the Q-points for the three diodes in Fig. 3.33 if R_1 is changed to 2.5 kΩ.

ANSWERS: (2.13 mA, 0.6 V); (1.13 mA, 0.6 V); (0 mA, −1.27 V)

EXERCISE: Use SPICE to calculate the Q-points of the diodes in the previous exercise. Use $I_S = 1$ fA.

ANSWERS: (2.12 mA, 0.734 V); (1.12 mA, 0.718 V); (0 mA, −1.19 V)

3.12 ANALYSIS OF DIODES OPERATING IN THE BREAKDOWN REGION

Reverse breakdown is actually a highly useful region of operation for the diode. The reverse breakdown voltage is nearly independent of current and can be used as either a voltage regulator or voltage reference. Thus, it is important to understand the analysis of diodes operating in reverse breakdown.

Figure 3.36 is a single-loop circuit containing a 20-V source supplying current to a Zener diode with a reverse breakdown voltage of 5 V. The voltage source has a polarity that will tend to reverse-bias the diode. Because the source voltage exceeds the Zener voltage rating of the diode, $V_Z = 5$ V, we should expect the diode to be operating in its breakdown region.

3.12.1 LOAD-LINE ANALYSIS

The *i*-*v* characteristic for this Zener diode is given in Fig. 3.37, and load-line analysis can be used to find the Q-point for the diode, independent of the region of operation. The normal polarities for I_D and V_D are indicated in Fig. 3.36, and the loop equation is

$$-20 = V_D + 5000 I_D \tag{3.36}$$

In order to draw the load line, we choose two points on the graph:

$$V_D = 0,\ I_D = -4 \text{ mA} \qquad \text{and} \qquad V_D = -5 \text{ V},\ I_D = -3 \text{ mA}$$

In this case the load line intersects the diode characteristic at a Q-point in the breakdown region: (−2.9 mA, −5.2 V).

3.12.2 ANALYSIS WITH THE PIECEWISE LINEAR MODEL

The assumption of reverse breakdown requires that the diode current I_D be less than zero or that the Zener current $I_Z = -I_D > 0$. We will analyze the circuit with the piecewise linear model and test this condition to see if it is consistent with the reverse-breakdown assumption.

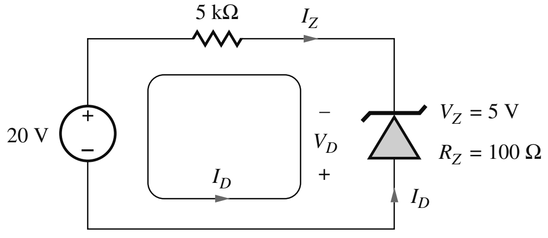


Figure 3.36 Circuit containing a Zener diode with $V_Z = 5$ V and $R_Z = 100\ \Omega$.

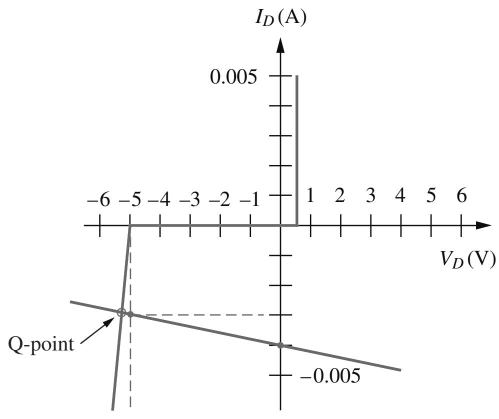


Figure 3.37 Load line for Zener diode.

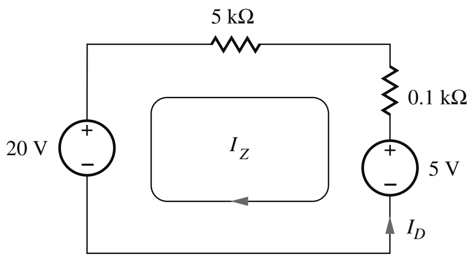


Figure 3.38 Circuit with piecewise linear model for Zener diode. Note that the diode model is valid only in the breakdown region of the characteristic.

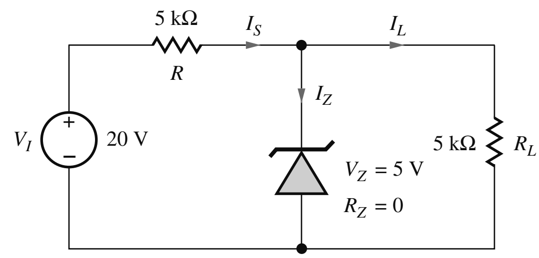


Figure 3.39 Zener diode voltage regulator circuit.

In Fig. 3.38, the Zener diode has been replaced with its piecewise linear model from Fig. 3.16 in Sec. 3.6, with $V_Z = 5$ V and $R_Z = 100\ \Omega$. Writing the loop equation this time in terms of I_Z:

$$20 - 5100 I_Z - 5 = 0 \qquad \text{or} \qquad I_Z = \frac{(20-5)\text{ V}}{5100\ \Omega} = 2.94 \text{ mA} \tag{3.37}$$

Because I_Z is greater than zero ($I_D < 0$), the solution is consistent with our assumption of Zener breakdown operation.

It is worth noting that diodes have three possible states when the breakdown region is included, (on, off, and reverse breakdown), further increasing analysis complexity.

3.12.3 VOLTAGE REGULATION

A useful application of the Zener diode is as a **voltage regulator,** as shown in the circuit of Fig. 3.39. The function of the Zener diode is to maintain a constant voltage across load resistor R_L. As long as the diode is operating in reverse breakdown, a voltage of approximately V_Z will appear across R_L. To ensure that the diode is operating in the Zener breakdown region, we must have $I_Z > 0$.

The circuit of Fig. 3.39 has been redrawn in Fig. 3.40 with the model for the Zener diode, with $R_Z = 0$. Using nodal analysis, the Zener current is expressed by $I_Z = I_I - I_L$. The currents I_I and I_L are equal to

$$I_I = \frac{V_I - V_Z}{R} = \frac{(20-5)\text{ V}}{5\text{ k}\Omega} = 3 \text{ mA} \qquad \text{and} \qquad I_L = \frac{V_Z}{R_L} = \frac{5\text{ V}}{5\text{ k}\Omega} = 1 \text{ mA} \tag{3.38}$$

resulting in a Zener current $I_Z = 2$ mA. $I_Z > 0$, which is again consistent with our assumptions. If the calculated value of I_Z were less than zero, then the Zener diode no longer controls the voltage across R_L, and the voltage regulator is said to have "dropped out of regulation."

For proper regulation to take place, the Zener current must be positive:

$$I_Z = I_I - I_L = \frac{V_I - V_Z}{R} - \frac{V_Z}{R_L} \geq 0 \tag{3.39}$$

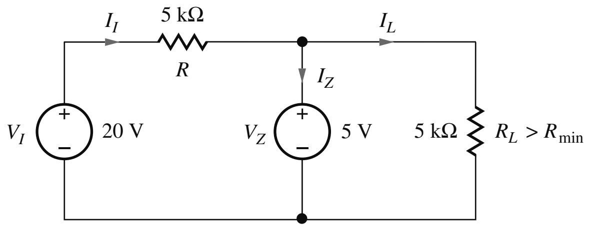


Figure 3.40 Circuit with a constant voltage model for the Zener diode.

Solving for R_L yields a lower bound on the value of load resistance for which the Zener diode will continue to act as a voltage regulator.

$$R_L > \frac{R}{\left(\dfrac{V_S}{V_Z} - 1\right)} = R_{\text{min}} \tag{3.40}$$

EXERCISE: What is the value of R_{min} for the Zener voltage regulator circuit in Figs. 3.39 and 3.40? What is the output voltage for $R_L = 1$ kΩ? For $R_L = 2$ kΩ?

ANSWERS: 1.67 kΩ; 3.33 V; 5.00 V

3.12.4 ANALYSIS INCLUDING ZENER RESISTANCE

The voltage regulator circuit in Fig. 3.39 has been redrawn in Fig. 3.41 and now includes a nonzero Zener resistance R_Z. The output voltage is now a function of the current I_Z through the Zener diode. For small values of R_Z, however, the change in output voltage will be small.

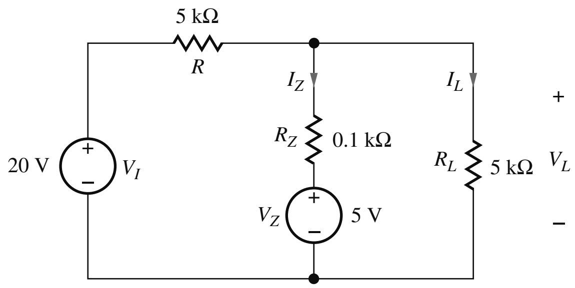


Figure 3.41 Zener diode regulator circuit, including Zener resistance.

EXAMPLE 3.9 DC ANALYSIS OF A ZENER DIODE REGULATOR CIRCUIT

Find the operating point for a Zener-diode-based voltage regulator circuit.

PROBLEM Find the output voltage and Zener diode current for the Zener diode regulator in Figs. 3.39 and 3.41 if $R_Z = 100\ \Omega$ and $V_Z = 5$ V.

SOLUTION **Known Information and Given Data:** Zener diode regulator circuit as modeled in Fig. 3.41 with $V_I = 20$ V, $R = 5$ kΩ, $R_Z = 0.1$ kΩ, and $V_Z = 5$ V

Unknowns: V_L, I_Z

Approach: The circuit contains a single unknown node voltage V_L, and a nodal equation can be written to find the voltage. Once V_L is found, I_Z can be determined using Ohm's law.

Assumptions: Use the piecewise linear model for the diode as drawn in Fig. 3.41.

Analysis: Writing the nodal equation for V_L yields

$$\frac{V_L - 20\text{ V}}{5000\ \Omega} + \frac{V_L - 5\text{ V}}{100\ \Omega} + \frac{V_L}{5000\ \Omega} = 0$$

Multiplying the equation by 5000 Ω and collecting terms gives

$$52V_L = 270 \text{ V} \qquad \text{and} \qquad V_L = 5.19 \text{ V}$$

The Zener diode current is equal to

$$I_Z = \frac{V_L - 5 \text{ V}}{100\ \Omega} = \frac{5.19 \text{ V} - 5 \text{ V}}{100\ \Omega} = 1.90 \text{ mA} > 0$$

Check of Results: $I_Z > 0$ confirms operation in reverse breakdown. We see that the output voltage of the regulator is slightly higher than for the case with $R_Z = 0$, and the Zener diode current is reduced slightly. Both changes are consistent with the addition of R_Z to the circuit.

Computer-Aided Analysis: We can use SPICE to simulate the Zener circuit if we specify the breakdown voltage using SPICE parameters BV, IBV, and RS. BV sets the breakdown voltage, and IBV represents the current at breakdown. Setting BV = 5 V and RS = 100 Ω and letting IBV default to 1 mA yields $V_L = 5.21$ V and $I_Z = 1.92$ mA, which agree well with our hand calculations. A transfer function analysis from V_S to V_L gives yields a sensitivity of 21 mV/V and an output resistance of 108 Ω. The meaning of these numbers is discussed in the next section.

EXERCISE: Find V_L, I_Z, and the Zener power dissipation in Fig. 3.41 if $R = 1$ kΩ.

ANSWERS: 6.25 V; 12.5 mA; 78.1 mW

3.12.5 LINE AND LOAD REGULATION

Two important parameters characterizing a voltage regulator circuit are **line regulation** and **load regulation.** Line regulation characterizes how sensitive the output voltage is to input voltage changes and is expressed as V/V or as a percentage. Load regulation characterizes how sensitive the output voltage is to changes in the load current withdrawn from the regulator and has the units of Ohms.

$$\text{Line regulation} = \frac{dV_L}{dV_I} \qquad \text{and} \qquad \text{Load regulation} = \frac{dV_L}{dI_L} \tag{3.41}$$

We can find expressions for these quantities from a straightforward analysis of the circuit in Fig. 3.41 similar to that in Ex. 3.9:

$$\frac{V_L - V_I}{R} + \frac{V_L - V_Z}{R_Z} + I_L = 0 \tag{3.42}$$

For a fixed load current, we find the line regulation as

$$\text{Line regulation} = \frac{R_Z}{R + R_Z} \tag{3.43}$$

and for changes in I_L as

$$\text{Load regulation} = -(R_Z \| R) \tag{3.44}$$

The load regulation should be recognized as the Thévenin equivalent resistance looking back into the regulator from the load terminals.

EXERCISE: What are the values of the load and line regulation for the circuit in Fig. 3.41?

ANSWERS: 19.6 mV/V; 98.0 Ω. Note that these are close to the SPICE results in Ex. 3.9.

3.13 HALF-WAVE RECTIFIER CIRCUITS

Rectifiers represent an application of diodes that we encounter frequently every day, but they may not be recognized as such. The basic **rectifier circuit** converts an ac voltage to a pulsating dc voltage. An LC or RC filter is then added to eliminate the ac components of the waveform and produce a nearly constant dc voltage output. Virtually every electronic device that is plugged into the wall utilizes a rectifier circuit to convert the 120-V, 60-Hz ac power line source to the various dc voltages required to operate electronic devices such as personal computers, audio systems, radio receivers, televisions, and the like. All of our battery chargers and "wall-warts" contain rectifiers. As a matter of fact, the vast majority of electronic circuits are powered by a dc source, usually based on some form of rectifier.

This section explores half-wave rectifier circuits with capacitor filters that form the basis for many dc power supplies. Up to this point, we have looked at only steady-state dc circuits in which the diode remained in one of its three possible states (on, off, or reverse breakdown). Now, however, the diode state will be changing with time, and a given piecewise linear model for the circuit will be valid for only a certain time interval.

3.13.1 HALF-WAVE RECTIFIER WITH RESISTOR LOAD

A single diode is used to form the **half-wave rectifier circuit** in Fig. 3.42. A sinusoidal voltage source $v_I = V_P \sin \omega t$ is connected to the series combination of diode D_1 and load resistor R. During the first half of the cycle, for which $v_I > 0$, the source forces a current through diode D_1 in the forward direction, and D_1 will be on. During the second half of the cycle, $v_I < 0$. Because a negative current cannot exist in the diode (unless it is in breakdown), it turns off. These two states are modeled in Fig. 3.43 using the ideal diode model.

When the diode is on, voltage source v_S is connected directly to the output and $v_O = v_I$. When the diode is off, the current in the resistor is zero, and the output voltage is zero. The input and output voltage waveforms are shown in Fig. 3.44(b), and the resulting current is called pulsating

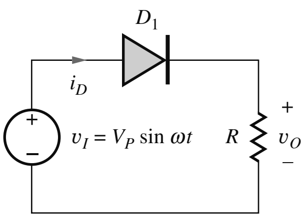


Figure 3.42 Half-wave rectifier circuit.

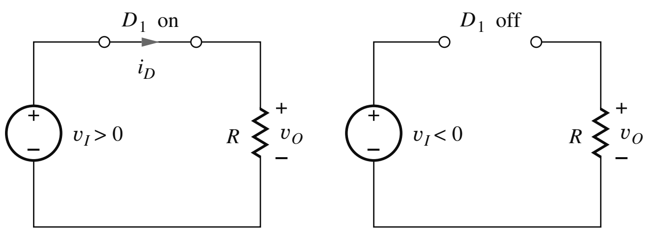


Figure 3.43 Ideal diode models for the two half-wave rectifier states.

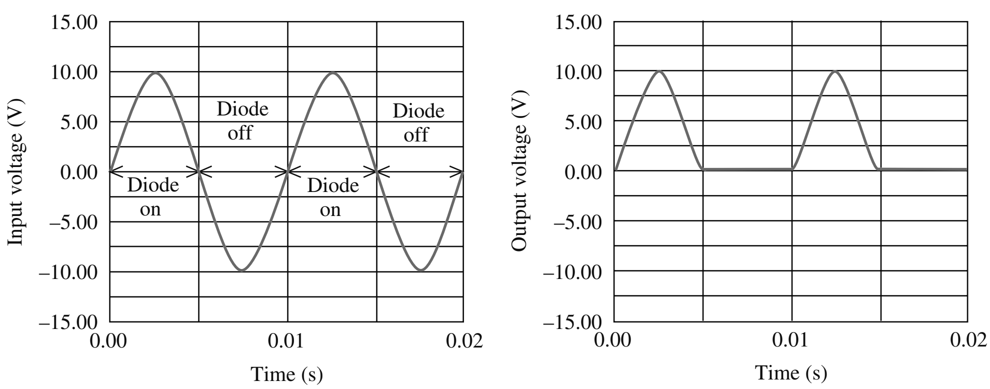


Figure 3.44 Sinusoidal input voltage v_S and pulsating dc output voltage v_O for the half-wave rectifier circuit.

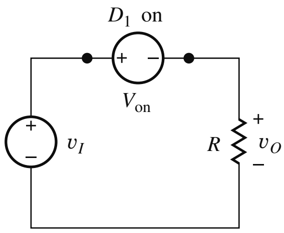


Figure 3.45 CVD model for the rectifier on state.

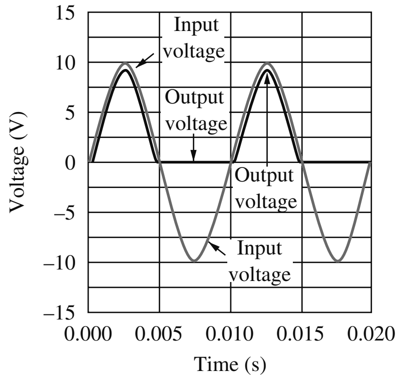


Figure 3.46 Half-wave rectifier output voltage with $V_P = 10$ V and $V_{on} = 0.7$ V.

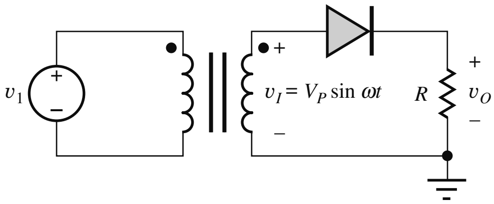


Figure 3.47 Transformer-driven half-wave rectifier.

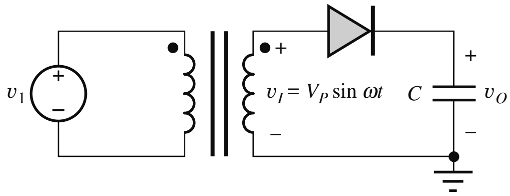


Figure 3.48 Rectifier with capacitor load (peak detector).

direct current. In this circuit, the diode is conducting 50 percent of the time and is off 50 percent of the time.

In some cases, the forward voltage drop across the diode can be important. Figure 3.45 shows the circuit model for the on-state using the CVD model. For this case, the output voltage is one diode-drop smaller than the input voltage during the conduction interval:

$$v_O = (V_P \sin \omega t) - V_{on} \qquad \text{for } V_P \sin \omega t \geq V_{on} \tag{3.45}$$

The output voltage remains zero during the off-state interval. The input and output waveforms for the half-wave rectifier, including the effect of V_{on}, are shown in Fig. 3.46 for $V_P = 10$ V and $V_{on} = 0.7$ V.

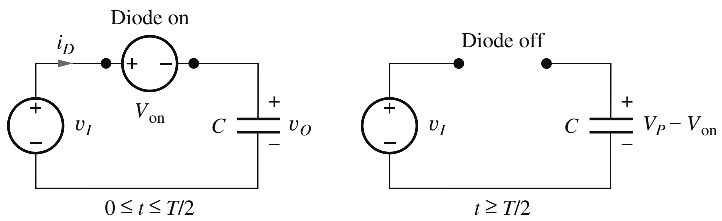
In many applications, a transformer is used to convert from the 120-V ac, 60-Hz voltage available from the power line to the desired ac voltage level, as in Fig. 3.47. The transformer can step the voltage up or down depending on the application; it also enhances safety by providing isolation from the power line. From circuit theory we know that the output of an ideal transformer can be represented by an ideal voltage source, and we use this knowledge to simplify the representation of subsequent rectifier circuit diagrams.

The unfiltered output of the half-wave rectifier in Fig. 3.42 or 3.47 is not suitable for operation of most electronic circuits because constant power supply voltages are required to establish proper bias for the electronic devices. A **filter capacitor** (or more complex circuit) can be added to filter the output of the circuit in Fig. 3.47 to remove the time-varying components from the waveform.

3.13.2 RECTIFIER FILTER CAPACITOR

To understand operation of the rectifier filter, we first consider operation of the **peak-detector** circuit in Fig. 3.48. This circuit is similar to that in Fig. 3.47 except that the resistor is replaced with a capacitor C that is initially discharged [$v_O(0) = 0$].

Models for the circuit with the diode in the on and off states are in Fig. 3.49, and the input and output voltage waveforms associated with this circuit are in Fig. 3.50. As the input voltage starts to



(a) (b)

Figure 3.49 Peak-detector circuit models (constant voltage drop model). (a) The diode is on for $0 \le t \le T/2$. (b) The diode is off for $t > T/2$.

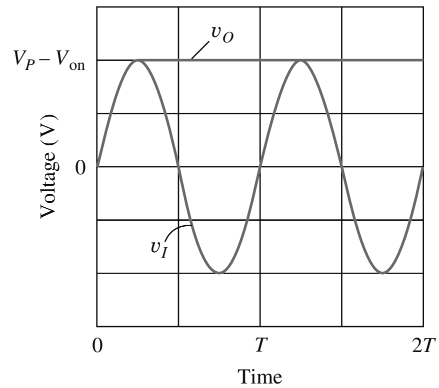


Figure 3.50 Input and output waveforms for the peak-detector circuit.

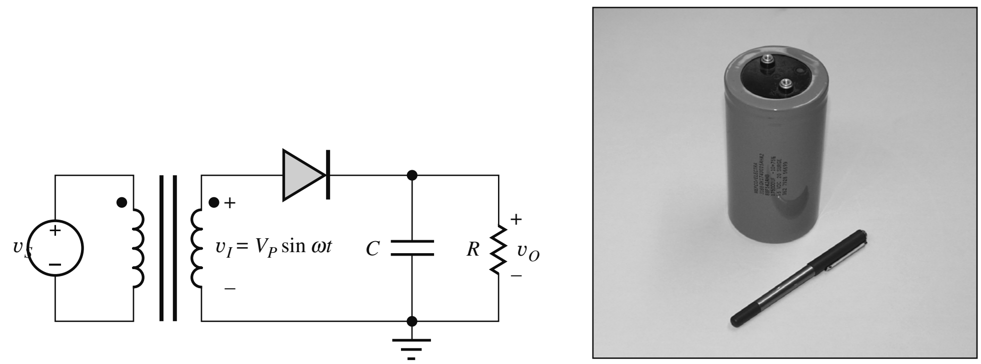
rise, the diode turns on and connects the capacitor to the source. The capacitor voltage equals the input voltage minus the voltage drop across the diode.

At the peak of the input voltage waveform, the current through the diode tries to reverse direction because $i_D = C[d(v_I - V_{on})/dt] < 0$, the diode cuts off, and the capacitor is disconnected from the rest of the circuit. There is no circuit path to discharge the capacitor, so the voltage on the capacitor remains constant. Because the amplitude of the input voltage source v_I can never exceed V_P, the capacitor remains disconnected from v_I for $t > T/2$. Thus, the capacitor in the circuit in Fig. 3.48 charges up to a voltage one diode-drop below the peak of the input waveform and then remains constant, thereby producing a dc output voltage

$$V_{dc} = V_P - V_{on} \tag{3.46}$$

3.13.3 HALF-WAVE RECTIFIER WITH *RC* LOAD

To make use of this output voltage, a load must be connected to the circuit as represented by the resistor R in Fig. 3.51. Now there is a path available to discharge the capacitor during the time the diode is not conducting. Models for the conducting and nonconducting time intervals are shown in Fig. 3.52; the waveforms for the circuit are shown in Fig. 3.53. The capacitor is again assumed to be



(a) (b)

Figure 3.51 (a) Half-wave rectifier circuit with filter capacitor; (b) a-175,000-μF, 15-V filter capacitor. Capacitance tolerance is −10 percent, +75 percent.

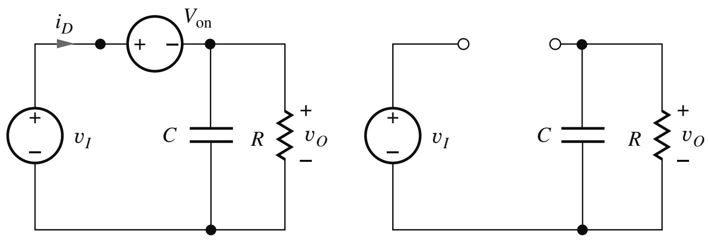


Figure 3.52 Half-wave rectifier circuit models.

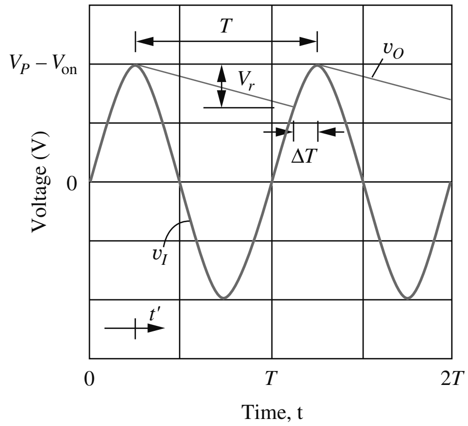


Figure 3.53 Input and output voltage waveforms for the half-wave rectifier circuit ($RC \gg T$).

initially discharged and the time constant RC is assumed to be $\gg T$. During the first quarter cycle, the diode conducts, and the capacitor is rapidly charged toward the peak value of the input voltage source. The diode cuts off at the peak of v_I, and the capacitor voltage then discharges exponentially through the resistor R, as governed by the circuit in Fig. 3.52(b). The discharge continues until the voltage $v_I - v_{on}$ exceeds the output voltage v_O, which occurs near the peak of the next cycle. The process is then repeated once every cycle.

3.13.4 RIPPLE VOLTAGE AND CONDUCTION INTERVAL

The output voltage is no longer constant as in the ideal peak-detector circuit but has a **ripple voltage** $\boldsymbol{V_r}$. In addition, the diode only conducts for a short time ΔT during each cycle. This time ΔT is called the **conduction interval,** and its angular equivalent is the **conduction angle** $\boldsymbol{\theta_c}$ where $\theta_c = \omega \Delta T$. The variables ΔT, θ_c, and V_r are important values related to dc power supply design, and we will now develop expressions for these parameters.

During the discharge period, the voltage across the capacitor is described by

$$v_o(t') = (V_P - V_{on}) \exp\left(-\frac{t'}{RC}\right) \qquad \text{for } t' = \left(t - \frac{T}{4}\right) \geq 0 \tag{3.47}$$

We have referenced the t' time axis to $t = T/4$ to simplify the equation. The ripple voltage V_r is given by

$$V_r = (V_P - V_{on}) - v_o(t') = (V_P - V_{on})\left[1 - \exp\left(-\frac{T - \Delta T}{RC}\right)\right] \tag{3.48}$$

A small value of V_r is desired in most power supply designs; a small value requires RC to be much greater than $T - \Delta T$. Using $\exp(-x) \cong 1 - x$ for small x results in an approximate expression for the ripple voltage:

$$V_r \cong (V_P - V_{on})\frac{T}{RC}\left(1 - \frac{\Delta T}{T}\right) \tag{3.49}$$

A small ripple voltage also means $\Delta T \ll T$, and the final simplified expression for the ripple voltage becomes

$$V_r \cong \frac{(V_P - V_{on})}{R}\frac{T}{C} = I_{dc}\frac{T}{C} \tag{3.50}$$

where

$$I_{dc} = \frac{V_P - V_{\text{on}}}{R} \tag{3.51}$$

The approximation of the exponential used in Eqs. (3.49) and (3.50) is equivalent to assuming that the capacitor is being discharged by a constant current so that the discharge waveform is a straight line. The ripple voltage V_R can be considered to be determined by an equivalent dc current discharging the capacitor C for a time period T (that is, $\Delta V = (I_{dc}/C)\,T$).

Approximate expressions can also be obtained for conduction angle θ_C and conduction interval ΔT. At time $t = \frac{5}{4}T - \Delta T$, the input voltage just exceeds the output voltage, and the diode is conducting. Therefore, $\theta = \omega t = 5\pi/2 - \theta_C$ and

$$V_p \sin\left(\frac{5}{2}\pi - \theta_C\right) - V_{\text{on}} = (V_P - V_{\text{on}}) - V_r \tag{3.52}$$

Remembering that $\sin(5\pi/2 - \theta_C) = \cos\theta_C$, we can simplify the above expression to

$$\cos\theta_C = 1 - \frac{V_r}{V_P} \tag{3.53}$$

For small values of θ_C, $\cos\theta_C \cong 1 - \theta_C^2/2$. Solving for the conduction angle and conduction interval gives

$$\theta_C = \sqrt{\frac{2V_r}{V_P}} \quad \text{and} \quad \Delta T = \frac{\theta_C}{\omega} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} \tag{3.54}$$

EXAMPLE 3.10 HALF-WAVE RECTIFIER ANALYSIS

Here we see an illustration of numerical results for a half-wave rectifier with a capacitive filter.

PROBLEM Find the value of the dc output voltage, dc output current, ripple voltage, conduction interval, and conduction angle for a half-wave rectifier driven from a transformer having a secondary voltage of 12.6 V_{rms} (60 Hz) with $R = 15\ \Omega$ and $C = 25{,}000\ \mu\text{F}$. Assume the diode on-voltage $V_{\text{on}} = 1$ V.

SOLUTION **Known Information and Given Data:** Half-wave rectifier circuit with RC load as depicted in Fig. 3.51; transformer secondary voltage is 12.6 V_{rms}, operating frequency is 60 Hz, $R = 15\ \Omega$, and $C = 25{,}000\ \mu\text{F}$.

Unknowns: dc output voltage V_{dc}, output current I_{dc}, ripple voltage V_r, conduction interval ΔT, conduction angle θ_C

Approach: Given data can be used directly to evaluate Eqs. (3.46), (3.50), (3.51), and (3.54).

Assumptions: Diode on-voltage is 1 V. Remember that the derived results assume the ripple voltage is much less than the dc output voltage ($V_r \ll V_{dc}$) and the conduction interval is much less than the period of the ac signal ($\Delta T \ll T$).

Analysis: The ideal dc output voltage in the absence of ripple is given by Eq. (3.46):

$$V_{dc} = V_P - V_{\text{on}} = \left(12.6\sqrt{2} - 1\right)\text{ V} = 16.8\text{ V}$$

The nominal dc current delivered by the supply is

$$I_{dc} = \frac{V_P - V_{\text{on}}}{R} = \frac{16.8 \text{ V}}{15 \ \Omega} = 1.12 \text{ A}$$

The ripple voltage is calculated using Eq. (3.50) with the discharge interval $T = 1/60$ s:

$$V_r \cong I_{dC}\frac{T}{C} = 1.12 \text{ A}\frac{\frac{1}{60} \text{ s}}{2.5 \times 10^{-2} \text{ F}} = 0.747 \text{ V}$$

The conduction angle is calculated using Eq. (3.54)

$$\theta_c = \omega \Delta T = \sqrt{\frac{2V_r}{V_P}} = \sqrt{\frac{2 \cdot 0.75}{17.8}} = 0.290 \text{ rad or } 16.6^\circ$$

and the conduction interval is

$$\Delta T = \frac{\theta_c}{\omega} = \frac{\theta_c}{2\pi f} = \frac{0.29}{120\pi} = 0.769 \text{ ms}$$

Check of Results: The ripple voltage represents 4.4 percent of the dc output voltage. Thus the assumption that the voltage is approximately constant is justified. The conduction time is 0.769 ms out of a total period $T = 16.7$ ms, and the assumption that $\Delta T \ll T$ is also satisfied.

Discussion: From this example, we see that even a 1-A power supply requires a significant filter capacitance C to maintain a low ripple percentage. In this case, $C = 0.025\text{F} = 25{,}000$ μF.

EXERCISE: Find the value of the dc output voltage, dc output current, ripple voltage, conduction interval, and conduction angle for a half-wave rectifier that is being supplied from a transformer having a secondary voltage of 6.3 V_{rms} (60 Hz) with $R = 0.5 \ \Omega$ and $C = 500{,}000$ μF. Assume the diode on voltage $V_{on} = 1$ V.

ANSWERS: 7.91 V; 15.8 A; 0.527 V; 0.912 ms; 19.7°

EXERCISE: What are the values of the dc output voltage and dc output current for a half-wave rectifier that is being supplied from a transformer having a secondary voltage of 10 V_{rms} (60 Hz) and a 2-Ω load resistor? Assume the diode on voltage $V_{on} = 1$ V. What value of filter capacitance is required to have a ripple voltage of no more than 0.1 V? What is the conduction angle?

ANSWERS: 13.1 V; 6.57 A; 1.10 F; 6.82°

3.13.5 DIODE CURRENT

In rectifier circuits, a nonzero current is present in the diode for only a very small fraction of the period T, yet an almost constant dc current is flowing out of the filter capacitor to the load. The total charge lost from the capacitor during each cycle must be replenished by the current through the diode during the short conduction interval ΔT, which leads to high peak diode currents. Figure 3.54 shows

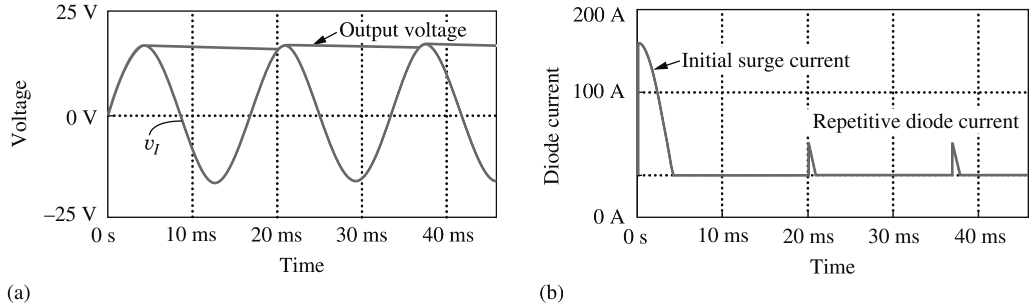


Figure 3.54 SPICE simulation of the half-wave rectifier circuit: (a) voltage waveforms; (b) diode current.

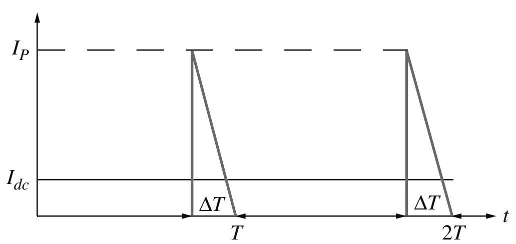


Figure 3.55 Triangular approximation to diode current pulse.

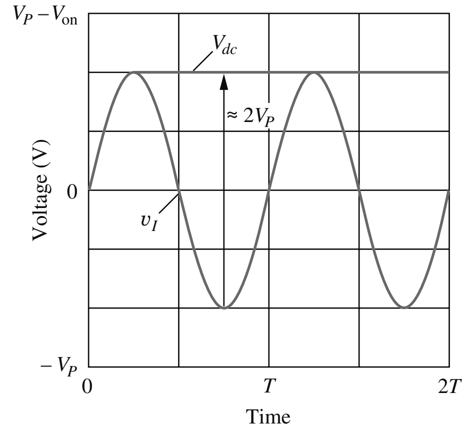


Figure 3.56 Peak reverse voltage across the diode in a half-wave rectifier.

the results of SPICE simulation of the diode current. The repetitive current pulse can be modeled approximately by a triangle of height I_P and width ΔT, as in Fig. 3.55.

Equating the charge supplied through the diode during the conduction interval to the charge lost from the filter capacitor during the complete period yields

$$Q = I_P \frac{\Delta T}{2} = I_{dc} T \qquad \text{or} \qquad I_P = I_{dc} \frac{2T}{\Delta T} \tag{3.55}$$

Here we remember that the integral of current over time represents charge Q. Therefore the charge supplied by the triangular current pulse in Fig. 3.55 is given by the area of the triangle, $I_P \Delta T/2$.

For Ex. 3.10, the peak diode current would be

$$I_P = 1.12 \frac{2 \cdot 16.7}{0.769} = 48.6 \text{ A} \tag{3.56}$$

which agrees well with the simulation results in Fig. 3.55. The diode must be built to handle these high peak currents, which occur over and over. This high peak current is also the reason for the relatively large choice of V_{on} used in Ex. 3.10. (See Prob. 3.82.)

EXERCISE: (a) What is the forward voltage of a diode operating at a current of 48.6 A at 300 K if $I_S = 10^{-15}$ A? (b) At 50 C?

ANSWERS: 0.994 V; 1.07 V

3.13.6 SURGE CURRENT

When the power supply is first turned on, the capacitor is completely discharged, and there will be an even larger current through the diode, as is visible in Fig. 3.54. During the first quarter cycle, the current through the diode is given approximately by

$$i_d(t) = i_c(t) \cong C\left[\frac{d}{dt}V_P \sin\omega t\right] = \omega C V_P \cos\omega t \tag{3.57}$$

The peak value of this initial **surge current** occurs at $t = 0^+$ and is given by

$$I_{SC} = \omega C V_P = 2\pi(60 \text{ Hz})(0.025 \text{ F})(17.8 \text{ V}) = 168 \text{ A}$$

Using the numbers from Ex. 3.10 yields an initial surge current of almost 170 A! This value, again, agrees well with the simulation results in Fig. 3.54. If the input signal v_I does not happen to be crossing through zero when the power supply is turned on, the situation can be even worse, and rectifier diodes selected for power supply applications must be capable of withstanding very large surge currents as well as the large repetitive current pulses required each cycle.

In most practical circuits, the surge current will be large but cannot actually reach the values predicted by Eq. (3.57) because of series resistances in the circuit that we have neglected. The rectifier diode itself will have an internal series resistance (review the SPICE model in Sec. 3.9 for example), and the transformer will have resistances associated with both the primary and secondary windings. A total series resistance in the secondary of only a few tenths of an ohm will significantly reduce both the surge current and peak repetitive current in the circuit. In addition, the large time constant associated with the series resistance and filter capacitance causes the rectifier output to take many cycles to reach its steady-state voltage. (See SPICE simulation problems at the end of this chapter.)

3.13.7 PEAK-INVERSE-VOLTAGE (PIV) RATING

We must also be concerned about the breakdown voltage rating of the diodes used in rectifier circuits. This breakdown voltage is called the **peak-inverse-voltage (PIV)** rating of the rectifier diode. The worst-case situation for the half-wave rectifier is depicted in Fig. 3.56 in which it is assumed that the ripple voltage V_r is very small. When the diode is off, as in Fig. 3.52(b), the largest reverse bias across the diode is equal to $V_{dc} - v_I$ which occurs when v_I reaches its negative peak of $-V_P$. The diode must therefore be able to withstand a reverse bias of at least

$$\text{PIV} \geq V_{dc} - v_I^{\min} = V_P - V_{\text{on}} - (-V_P) = 2V_P - V_{\text{on}} \cong 2V_P \tag{3.58}$$

From Eq. (3.58), we see that diodes used in the half-wave rectifier circuit must have a PIV rating equal to twice the peak voltage supplied by the source v_I. The PIV value corresponds to the minimum value of Zener breakdown voltage for the rectifier diode. A safety margin of at least 25 to 50 percent is usually specified for the diode PIV rating in power supply designs.

3.13.8 DIODE POWER DISSIPATION

In high-current power supply applications, the power dissipation in the rectifier diodes can become significant. The average power dissipation in the diode is defined by

$$P_D = \frac{1}{T}\int_0^T v_D(t)\,i_D(t)\,dt \tag{3.59}$$

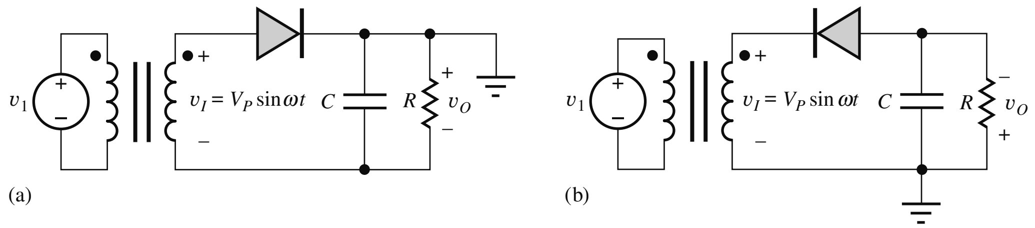


Figure 3.57 Half-wave rectifier circuits that develop negative output voltages.

This expression can be simplified by assuming that the voltage across the diode is approximately constant at $v_D(t) = V_{\text{on}}$ and by using the triangular approximation to the diode current $i_D(t)$ shown in Fig. 3.55. Eq. (3.59) becomes

$$P_D = \frac{1}{T}\int_0^T V_{\text{on}} i_D(t)\,dt = \frac{V_{\text{on}}}{T}\int_{T-\Delta T}^{T} i_D(t)\,dt = V_{\text{on}}\frac{I_P}{2}\frac{\Delta T}{T} = V_{\text{on}} I_{dc} \tag{3.60}$$

Using Eq. (3.55) we see that the power dissipation is equivalent to the constant dc output current multiplied by the on-voltage of the diode. For the half-wave rectifier example, $P_D = (1\text{ V})(1.1\text{ A}) = 1.1$ W. This rectifier diode would probably need a heat sink to maintain its temperature at a reasonable level. Note that the average current through the diode is I_{dc}.

Another source of power dissipation is caused by resistive loss within the diode. Diodes have a small internal series resistance R_S, and the average power dissipation in this resistance can be calculated using

$$P_D = \frac{1}{T}\int_0^T i_D^2(t) R_S\,dt \tag{3.61}$$

Evaluation of this integral (left for Prob. 3.87) for the triangular current wave form in Fig. 3.55 yields

$$P_D = \frac{1}{3} I_P^2 R_S \frac{\Delta T}{T} = \frac{4}{3}\frac{T}{\Delta T} I_{dc}^2 R_S \tag{3.62}$$

Using the number from the rectifier example with $R_S = 0.20\ \Omega$ yields $P_D = 7.3$ W! This is significantly greater than the component of power dissipation caused by the diode on-voltage calculated using Eq. (3.60). The component of power dissipation described by Eq. (3.62) can be reduced by minimizing the peak current I_P through the use of the minimum required size of filter capacitor or by using the full-wave rectifier circuits, which are discussed in Sec. 3.14.

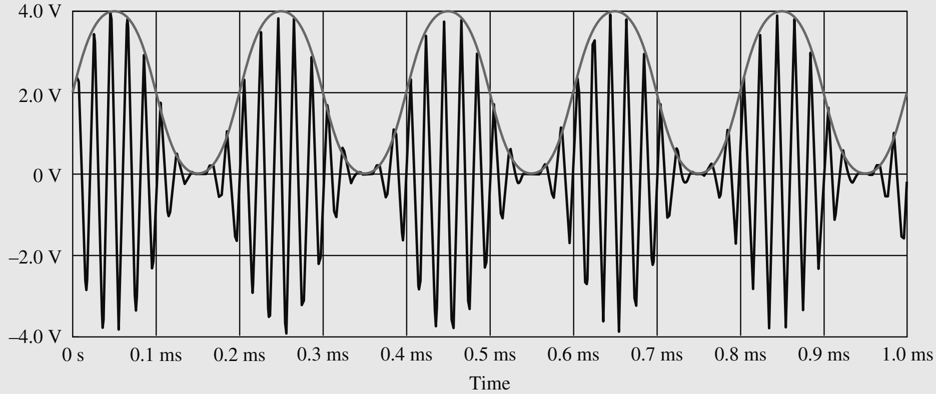
3.13.9 HALF-WAVE RECTIFIER WITH NEGATIVE OUTPUT VOLTAGE

The circuit of Fig. 3.51 can also be used to produce a negative output voltage if the top rather than the bottom of the capacitor is grounded, as depicted in Fig. 3.57(a) or by reversing the direction of the diode in the original circuit as in Fig. 3.57(b). These two circuits are equivalent. In the circuit in Fig. 3.57(b), the diode conducts on the negative half cycle of the transformer voltage v_I, and the dc output voltage is $V_{dc} = -(V_P - V_{\text{on}})$.

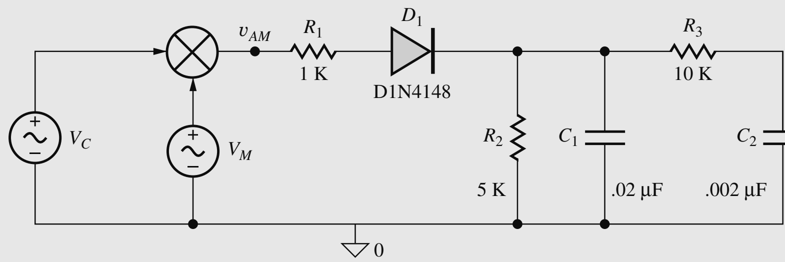
ELECTRONICS IN ACTION

AM Demodulation

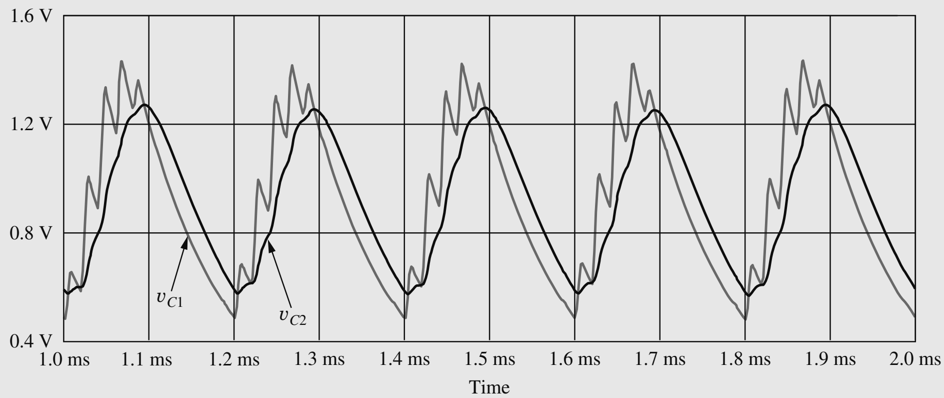
The waveform for a 100 percent amplitude modulated (AM) signal is shown in the figure below and described mathematically by $v_{AM} = 2 \sin \omega_C t(1 + \sin \omega_M t)$ V in which ω_C is the carrier frequency ($f_C = 50$ kHz) and ω_M is the modulating frequency ($f_M = 5$ kHz). The envelope of



the AM signal contains the information being transmitted, and the envelope can be recovered from the signal using a simple half-wave rectifier. In the SPICE circuit below, the signal to be demodulated is applied as the input signal to the rectifier, and the rectifier, and the R_2C_1 time



constant is set to filter out the carrier frequency but follow the signal's envelope. Additional filtering is provided by the low-pass filter formed by R_3 and C_2. SPICE simulation results appear below along with the results of a Fourier analysis of the demodulated signal. The plots of v_{C1} and v_{C2} represent the voltages across capacitors C_1 and C_2 respectively.



SPICE Results for Spectral Content of v_{C2} (V)

5 kHz	0.330
10 kHz	0.046
15 kHz	0.006
20 kHz	0.001
–	
45 kHz	0.006
50 kHz	0.007
55 kHz	0.004

3.14 FULL-WAVE RECTIFIER CIRCUITS

Full-wave rectifier circuits cut the capacitor discharge time in half and offer the advantage of requiring only one-half the filter capacitance to achieve a given ripple voltage. The full-wave rectifier circuit in Fig. 3.58 uses a **center-tapped transformer** to generate two voltages that have equal amplitudes but are 180 degrees out of phase. With voltage v_I applied to the anode of D_1, and $-v_I$ applied to the anode of D_2, the two diodes form a pair of half-wave rectifiers operating on alternate half cycles of the input waveform. Proper phasing is indicated by the dots on the two halves of the transformer.

For $v_I > 0$, D_1 will be functioning as a half-wave rectifier, and D_2 will be off, as indicated in Fig. 3.59. The current exits the upper terminal of the transformer, goes through diode D_1, through the RC load, and returns back into the center tap of the transformer.

For $v_I < 0$, D_1 will be off, and D_2 will be functioning as a half-wave rectifier as indicated in Fig. 3.60. During this portion of the cycle, the current path leaves the bottom terminal of the transformer, goes through D_2, down through the RC load, and again returns into the transformer center tap. The current direction in the load is the same during both halves of the cycle; one-half of the transformer is utilized during each half cycle.

The load, consisting of the filter capacitor C and load resistor R, now receives two current pulses per cycle, and the capacitor discharge time is reduced to less than $T/2$, as indicated in the graph in Fig. 3.61. An analysis similar to that for the half-wave rectifier yields the same formulas for dc output voltage, ripple voltage, and ΔT, except that the discharge interval is $T/2$ rather than T. For a given capacitor value, the ripple voltage is one-half as large, and the conduction interval and peak

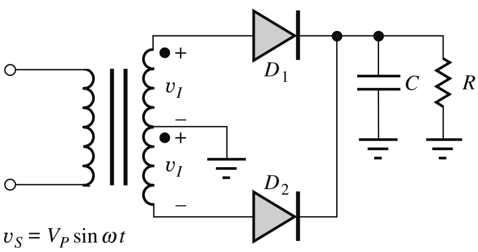


Figure 3.58 Full-wave rectifier circuit using two diodes and a center-tapped transformer. This circuit produces a positive output voltage.

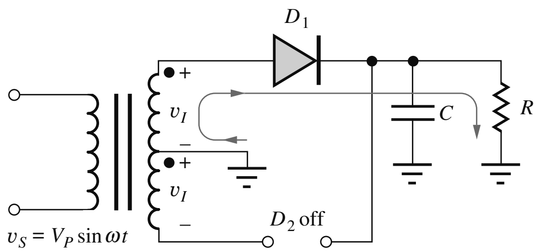


Figure 3.59 Equivalent circuit for $v_I > 0$.

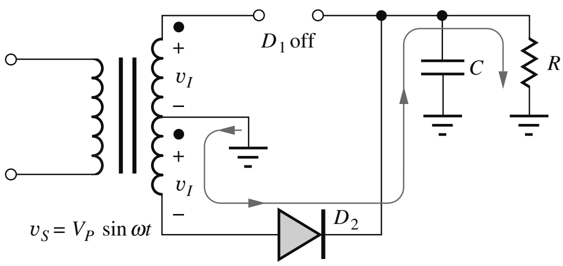


Figure 3.60 Equivalent circuit for $v_I < 0$.

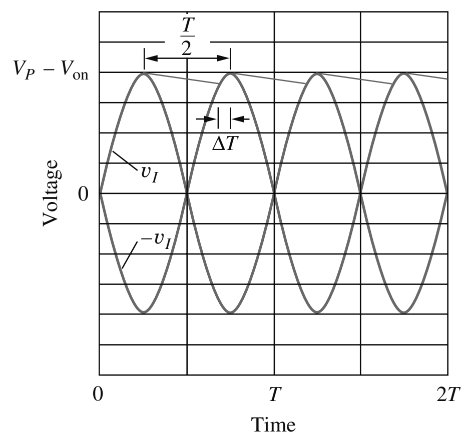


Figure 3.61 Voltage waveforms for the full-wave rectifier.

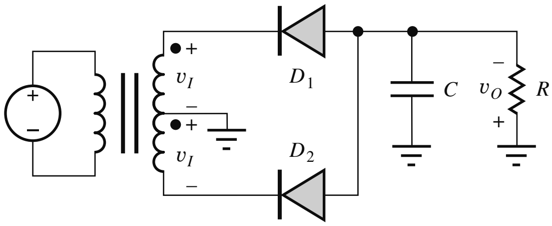


Figure 3.62 Full-wave rectifier with negative output voltage.

current are reduced. The peak-inverse-voltage waveform for each diode is similar to the one shown in Fig. 3.56 for the half-wave rectifier, with the result that the PIV rating of each diode is the same as in the half-wave rectifier.

3.14.1 FULL-WAVE RECTIFIER WITH NEGATIVE OUTPUT VOLTAGE

By reversing the polarity of the diodes, as in Fig. 3.62, a full-wave rectifier circuit with a negative output voltage is realized. Other aspects of the circuit remain the same as the previous full-wave rectifiers with positive output voltages.

3.15 FULL-WAVE BRIDGE RECTIFICATION

The requirement for a center-tapped transformer in the full-wave rectifier can be eliminated through the use of two additional diodes in the **full-wave bridge rectifier circuit** configuration shown in Fig. 3.63. For $v_I > 0$, D_2 and D_4 will be on and D_1 and D_3 will be off, as indicated in Fig. 3.64. Current exits the top of the transformer, goes through D_2 into the RC load, and returns to the transformer through D_4. The full transformer voltage, now minus two diode voltage drops, appears across the load capacitor yielding a dc output voltage

$$V_{dc} = V_P - 2V_{\text{on}} \tag{3.63}$$

The peak voltage at node 1, which represents the maximum reverse voltage appearing across D_1, is equal to $(V_P - V_{\text{on}})$. Similarly, the peak reverse voltage across diode D_3 is $(V_P - 2V_{\text{on}}) - (-V_{\text{on}}) = (V_P - V_{\text{on}})$.

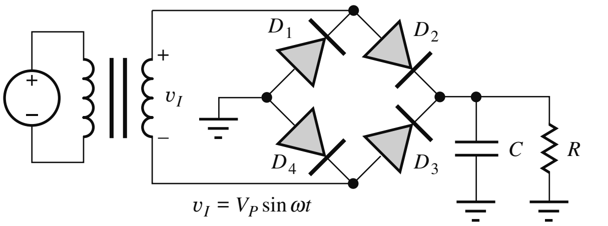


Figure 3.63 Full-wave bridge rectifier circuit with positive output voltage.

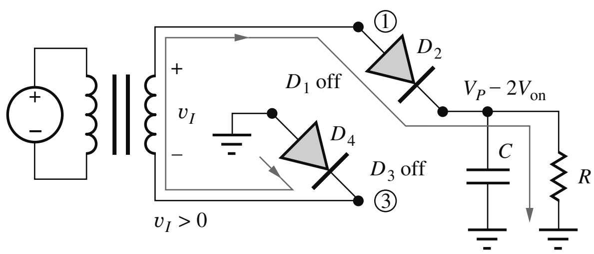


Figure 3.64 Full-wave bridge rectifier circuit for $v_I > 0$.

For $v_I < 0$, D_1 and D_3 will be on and D_2 and D_4 will be off, as depicted in Fig. 3.65. Current leaves the bottom of the transformer, goes through D_3 into the RC load, and back through D_1 to the transformer. The full transformer voltage is again being utilized. The peak voltage at node 3 is now equal to $(V_P - V_{\text{on}})$ and is the maximum reverse voltage appearing across D_4. Similarly, the peak reverse voltage across diode D_2 is $(V_P - 2V_{\text{on}}) - (-V_{\text{on}}) = (V_P - V_{\text{on}})$.

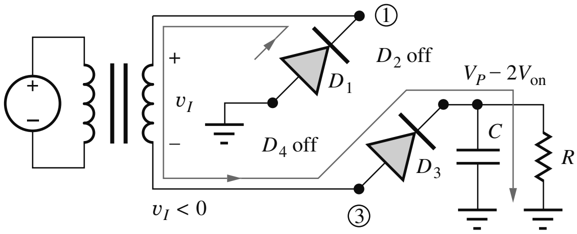


Figure 3.65 Full-wave bridge rectifier circuit for $v_I < 0$.

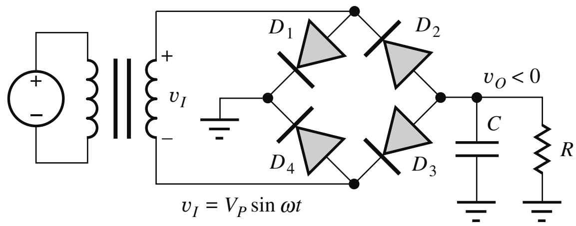


Figure 3.66 Full-wave bridge rectifier circuit with $v_O < 0$.

From the analysis of the two half cycles, we see that each diode must have a PIV rating given by

$$\text{PIV} = V_P - V_{\text{on}} \cong V_P \tag{3.64}$$

As with the previous rectifier circuits, a negative output voltage can be generated by reversing the direction of the diodes, as in the circuit in Fig. 3.66.

3.16 RECTIFIER COMPARISON AND DESIGN TRADEOFFS

Tables 3.5 and 3.6 summarize the characteristics of the half-wave, full-wave, and full-wave bridge rectifiers introduced in Secs. 3.13 to 3.15. The filter capacitor often represents a significant economic factor in terms of cost, size, and weight in the design of **rectifier circuits.** For a given ripple voltage, the value of the filter capacitor required in the full-wave rectifier is one-half that for the half-wave rectifier.

The reduction in peak current in the full-wave rectifier can significantly reduce heat dissipation in the diodes. The addition of the second diode and the use of a center-tapped transformer represent additional expenses that offset some of the advantage. However, the benefits of full-wave rectification usually outweigh the minor increase in circuit complexity.

The bridge rectifier eliminates the need for the center-tapped transformer, and the PIV rating of the diodes is reduced, which can be particularly important in high-voltage circuits. The cost of the extra diodes is usually negligible, particularly since four-diode bridge rectifiers can be purchased in single-component form.

TABLE 3.5
Rectifier Equation Summary

HALF-WAVE RECTIFIER	FULL-WAVE RECTIFIER	FULL-WAVE BRIDGE RECTIFIER
$V_{dc} = V_P - V_{\text{on}} \quad I_{dc} = \dfrac{(V_P - V_{\text{on}})}{R}$	$V_{dc} = V_P - V_{\text{on}} \quad I_{dc} = \dfrac{(V_P - V_{\text{on}})}{R}$	$V_{dc} = V_P - 2V_{\text{on}} \quad I_{dc} = \dfrac{(V_P - 2V_{\text{on}})}{R}$
$V_r = \dfrac{(V_P - V_{\text{on}})}{R}\dfrac{T}{C} = I_{dc}\dfrac{T}{C}$	$V_r = \dfrac{(V_P - V_{\text{on}})}{R}\dfrac{T}{2C} = I_{dc}\dfrac{T}{2C}$	$V_r = \dfrac{(V_P - 2V_{\text{on}})}{R}\dfrac{T}{2C} = I_{dc}\dfrac{T}{2C}$
$\Delta T = \dfrac{1}{\omega}\sqrt{\dfrac{2V_r}{V_P}} \quad \theta_c = \omega\Delta T$	$\Delta T = \dfrac{1}{\omega}\sqrt{\dfrac{2V_r}{V_P}} \quad \theta_c = \omega\Delta T$	$\Delta T = \dfrac{1}{\omega}\sqrt{\dfrac{2V_r}{V_P}} \quad \theta_c = \omega\Delta T$
$I_P = I_{dc}\dfrac{2T}{\Delta T} \quad \text{PIV} = 2V_P$	$I_P = I_{dc}\dfrac{T}{\Delta T} \quad \text{PIV} = 2V_P$	$I_P = I_{dc}\dfrac{T}{\Delta T} \quad \text{PIV} = V_P$

TABLE 3.6
Comparison of Rectifiers with Capacitive Filters

RECTIFIER PARAMETER	HALF-WAVE RECTIFIER	FULL-WAVE RECTIFIER	FULL-WAVE BRIDGE RECTIFIER
Filter capacitor	C	$\frac{C}{2}$	$\frac{C}{2}$
PIV rating	$2V_P$	$2V_P$	V_P
Peak diode current (constant V_r)	Highest I_P	Reduced $\frac{I_P}{2}$	Reduced $\frac{I_P}{2}$
Surge current	Highest	Reduced ($\propto C$)	Reduced ($\propto C$)
Comments	Least complexity	Smaller capacitor Requires center-tapped transformer Two diodes	Smaller capacitor Four diodes No center tap on transformer

DESIGN EXAMPLE 3.11 RECTIFIER DESIGN

Now we will use our rectifier theory to design a rectifier circuit that will provide a specified output voltage and ripple voltage.

PROBLEM Design a rectifier to provide a dc output voltage of 15 V with no more than 1 percent ripple at a load current of 2 A.

SOLUTION **Known Information and Given Data:** $V_{dc} = 15$ V, $V_r < 0.15$ V, $I_{dc} = 2$ A

Unknowns: Circuit topology, transformer voltage, filter capacitor, diode PIV rating, diode repetitive current rating, diode surge current rating.

Approach: Use given data to evaluate rectifier circuit equations. Let us choose a full-wave bridge topology that requires a smaller value of filter capacitance, a smaller diode PIV voltage, and no center tap in the transformer.

Assumptions: Assume diode on-voltage is 1 V. The ripple voltage is much less than the dc output voltage ($V_r \ll V_{dc}$), and the conduction interval should be much less than the period of the ac signal ($\Delta T \ll T$).

Analysis: The required transformer voltage is

$$V = \frac{V_P}{\sqrt{2}} = \frac{V_{dc} + 2V_{\text{on}}}{\sqrt{2}} = \frac{15 + 2}{\sqrt{2}}\text{ V} = 12.0\text{ V}_{\text{rms}}$$

The filter capacitor is found using the ripple voltage, output current, and discharge interval:

$$C = I_{dc}\left(\frac{T/2}{V_r}\right) = 2\text{ A}\left(\frac{1}{120}\text{ s}\right)\left(\frac{1}{0.15\text{ V}}\right) = 0.111\text{ F}$$

To find I_P, the conduction time is calculated using Eq. (3.54)

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}} = \frac{1}{120\pi}\sqrt{\frac{2(0.15)\text{ V}}{17\text{ V}}} = 0.352\text{ ms}$$

and the peak repetitive current is found to be

$$I_P = I_{dc}\left(\frac{2}{\Delta T}\right)\left(\frac{T}{2}\right) = 2\text{ A}\frac{(1/60)\text{ s}}{0.352\text{ ms}} = 94.7\text{ A}$$

The surge current estimate is

$$I_{\text{surge}} = \omega C V_P = 120\pi(0.111)(17) = 711\text{ A}$$

The minimum diode PIV is $V_P = 17$ V. A choice with a safety margin would be PIV > 20 V. The repetitive current rating should be 95 A with a surge current rating of 710 A. Note that both of these calculations overestimate the magnitude of the currents because we have neglected series resistance of the transformer and diode. The minimum filter capacitor needs to be 111,000 μF. Assuming a tolerance of −30 percent, a nominal filter capacitance of 160,000 μF would be required.

Check of Results: The ripple voltage is designed to be 1 percent of the dc output voltage. Thus the assumption that the voltage is approximately constant is justified. The conduction time is 0.352 ms out of a total period $T = 16.7$ ms. Thus the assumption that $\Delta T \ll T$ is satisfied.

Computer-Aided Analysis: This design example represents an excellent place where simulation can be used to explore the magnitude of the diode currents and improve the design so that we don't over-specify the rectifier diodes. A SPICE simulation with $R_S = 0.1\ \Omega$, $n = 2$, $I_S = 1\ \mu$A, and a transformer series resistance of 0.1 Ω yields a number of unexpected results: $I_P = 11$ A, $I_{\text{surge}} = 70$ A, and $V_{dc} = 13$ V! The surge current and peak repetitive current are both reduced by almost an order of magnitude compared to our hand calculations! In addition the output voltage is lower than expected. If we think further, a peak current of 11 A will cause a peak voltage drop of 2.2 V across the total series resistance of 0.2 Ω, so it should not be surprising that the output voltage is 2 V lower than originally expected. The series resistances actually help to reduce the stress on the diodes. The time constant of the series resistance and the filter capacitor is 0.44 s, so the circuit takes many cycles to reach the steady-state output voltage.

EXERCISE: Repeat the rectifier design assuming the use of a half-wave rectifier.

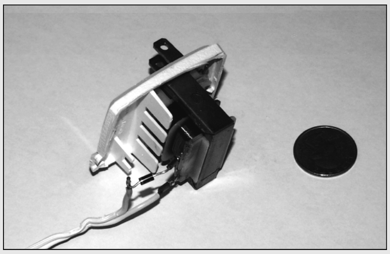
ANSWERS: $V = 11.3\ V_{rms}$; $C = 222{,}000\ \mu$F; $I_P = 184$ A; $I_{SC} = 1340$ A

ELECTRONICS IN ACTION

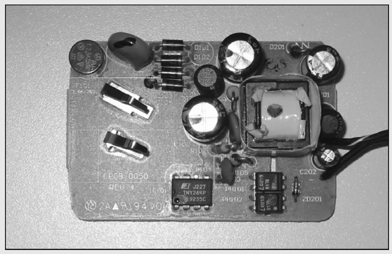
Power Cubes and Cell Phone Chargers

We actually encounter the unfiltered transformer driven half-wave rectifier circuit depicted in Fig. 3.47 frequently in our everyday lives in the form of "power cubes" and battery chargers for many portable electronic devices. An example is shown in the accompanying figure. The power cube contains only a small transformer and rectifier diode. The transformer is wound with small wire and has a significant resistance in both the primary and secondary windings. In the transformer in the photograph, the primary resistance is 600 Ω and the secondary resistance is 15 Ω, and these resistances actually help provide protection from failure of the transformer windings. Load resistance R in Fig. 3.51 represents the actual electronic device that is receiving power from the power cube and may often be a rechargable battery. In some cases, a filter capacitor may be included as part of the circuit that forms the load for the power cube.

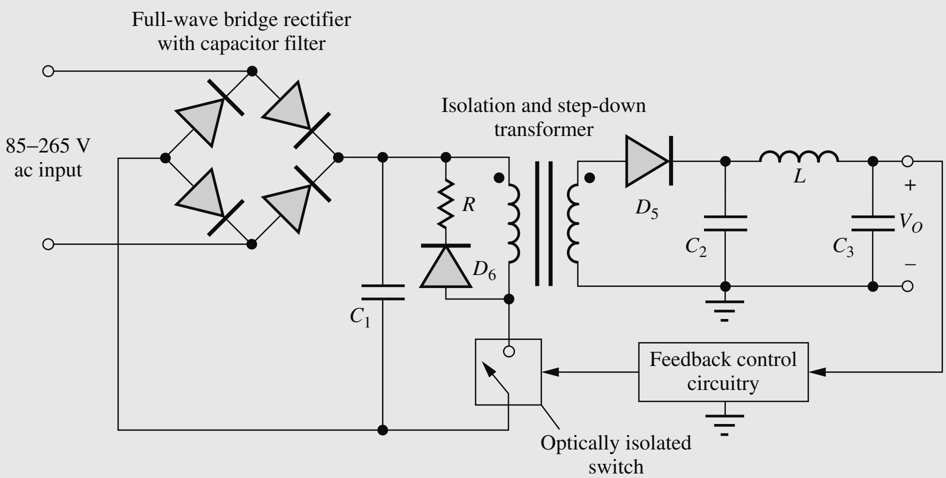
Part (c) of the figure below shows a much more complex device used for recharging the batteries in a cell phone. The simplified schematic in part (c) utilizes a full-wave bridge rectifier with filter capacitor connected directly to the ac line. The rectifier's high voltage output is filtered by capacitor C_1 and feeds a switching regulator consisting of a switch, the transformer driving a half-wave rectifier with pi-filter (D_5, C_2, L, and C_3), and a feedback circuit that controls the output voltage by modulating the duty cycle of the switch. The transformer steps down the voltage and provides isolation from the high voltage ac line input. Diode D_6 and R clamp the inductor voltage when the switch opens. The feedback signal path is isolated from the input using an optical isolator. (See Electronics in Action in Chapter 5 for discussion of an optical isolator.) Note the wide range of input voltages accomodated by the circuit. Thus, most international voltage standards can be accommodated by one adopter.



(a)



(b)



(c)

(a) Inside a simple power cube; (b) cell phone charger; (c) simplified schematic for the cell phone charger.

3.17 DYNAMIC SWITCHING BEHAVIOR OF THE DIODE

Up to this point, we have tacitly assumed that diodes can turn on and off instantaneously. However, an unusual phenomenon characterizes the dynamic switching behavior of the *pn* junction diode. SPICE simulation is used to illustrate the switching of the diode in the circuit in Fig. 3.67, in which diode D_1 is being driven from voltage source v_I through resistor R_1.

The source is zero for $t < 0$. At $t = 0$, the source voltage rapidly switches to $+1.5$ V, forcing a current into the diode to turn it on. The voltage remains constant until $t = 7.5$ ns. At this point the source switches to -1.5 V in order to turn the diode back off.

The simulation results are presented in Fig. 3.68. Following the voltage source change at $t = 0+$, the current increases rapidly, but the internal capacitance of the diode prevents the diode voltage from changing instantaneously. The current actually overshoots its final value and then decreases as the diode turns on and the diode voltage increases to approximately 0.7 V. At any given time, the current flowing into the diode is given by

$$i_D(t) = \frac{v_1(t) - v_D(t)}{0.75\ \text{k}\Omega} \tag{3.65}$$

The initial peak of the current occurs when v_I reaches 1.5 V and v_D is still nearly zero:

$$i_{D\max} = \frac{1.5\ \text{V}}{0.75\ \text{k}\Omega} = 2.0\ \text{mA} \tag{3.66}$$

After the diode voltage reaches its final value with $V_{\text{on}} \approx 0.7$ V, the current stabilizes at a forward current I_F of

$$I_F = \frac{1.5 - 0.7}{0.75\ \text{k}\Omega} = 1.1\ \text{mA} \tag{3.67}$$

At $t = 7.5$ ns, the input source rapidly changes polarity to -1.5 V, and a surprising thing happens. The diode current also rapidly reverses direction and is much greater than the reverse saturation current of the diode! The diode does not turn off immediately. In fact, the diode actually

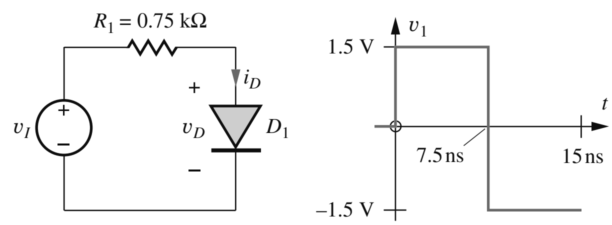


Figure 3.67 Circuit used to explore diode-switching behavior.

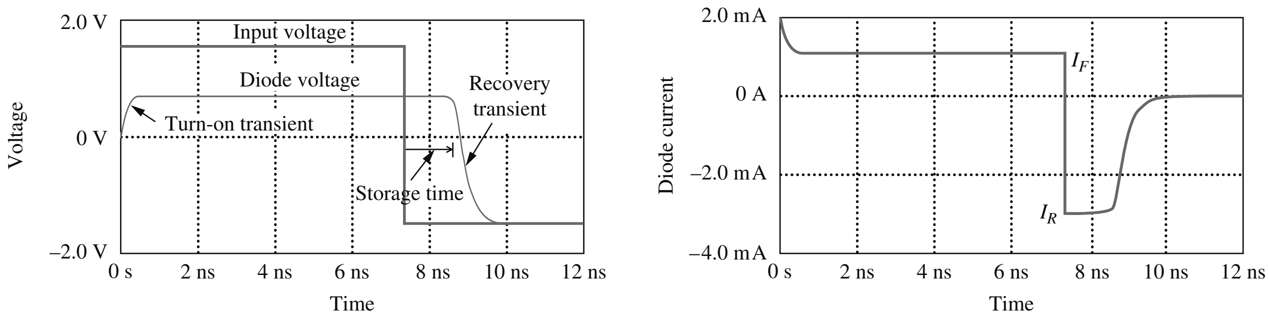


Figure 3.68 SPICE simulation results for the diode circuit in Fig. 3.67. (The diode transit time is equal to 5 ns.)

remains forward-biased by the charge stored in the diode, with $v_D = V_{\text{on}}$, even though the current has changed direction! The reverse current I_R is equal to

$$I_R = \frac{-1.5 - 0.7}{0.75\ \text{k}\Omega} = -2.9\ \text{mA} \tag{3.68}$$

The current remains at −2.9 mA for a period of time called the diode **storage time τ_S,** during which the internal charge stored in the diode is removed. Once the stored charge has been removed, the voltage across the diode begins to drop and charges toward the final value of −1.5 V. The current in the diode drops rapidly to zero as the diode voltage begins to fall.

The turn-on time and recovery time are determined primarily by the charging and discharging of the nonlinear depletion-layer capacitance C_j through the resistance R_S. The storage time is determined by the diffusion capacitance and diode transit time defined in Eq. (3.22) and by the values of the forward and reverse currents I_F and I_R:

$$\tau_S = \tau_T \ \ln\left[1 - \frac{I_F}{I_R}\right] = 5 \ln\left[1 - \frac{1.1\ \text{mA}}{-2.9\ \text{mA}}\right]\ \text{ns} = 1.6\ \text{ns} \tag{3.69}$$

The SPICE simulation results in Fig. 3.68 agree well with this value.

Always remember that solid-state devices do not turn off instantaneously. The unusual storage time behavior of the diode is an excellent example of the switching delays that occur in *pn* junction devices in which carrier flow is dominated by the minority-carrier diffusion process. This behavior is not present in field-effect transistors, in which current flow is dominated by majority-carrier drift.

3.18 PHOTO DIODES, SOLAR CELLS, AND LIGHT-EMITTING DIODES

Several other important applications of diodes include photo detectors in communication systems, solar cells for generating electric power, and light-emitting diodes (LEDs). These applications all rely on the solid-state diode's ability to detect on produce optical emissions.

3.18.1 PHOTO DIODES AND PHOTODETECTORS

If the depletion region of a *pn* junction diode is illuminated with light of sufficiently high energy, the photons can cause electrons to jump the semiconductor bandgap, creating electron–hole pairs. For photon absorption to occur, the incident photons must have an energy E_p that exceeds the bandgap of the semiconductor:

$$E_p = h\nu = \frac{hc}{\lambda} \geq E_G \tag{3.70}$$

where h = Planck's constant (6.626×10^{-34} J · s) λ = wavelength of optical illumination
ν = frequency of optical illumination c = velocity of light (3×10^8 m/s)

The *i-v* characteristic of a diode with and without illumination is shown in Fig. 3.69. The original diode characteristic is shifted vertically downward by the photon-generated current. Photon absorption creates an additional current crossing the *pn* junction that can be modeled by a current source i_{PH} in parallel with the *pn* junction diode, as shown in Fig. 3.70.

Based on this model, we see that the incident optical signal can be converted to an electrical signal voltage using the simple **photodetector circuit** in Fig. 3.71. The diode is reverse-biased to enhance the width and electric field in the depletion region. The photon-generated current i_{PH} will flow through resistor R and produce an output signal voltage given by

$$v_o = i_{\text{PH}} R \tag{3.71}$$

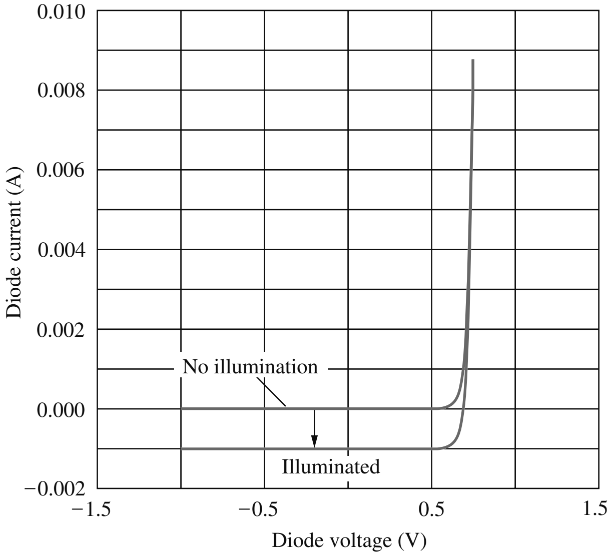


Figure 3.69 Diode *i-v* characteristic with and without optical illumination.

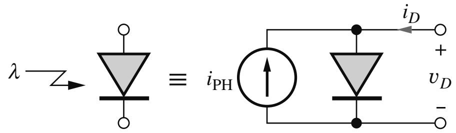


Figure 3.70 Model for optically illuminated diode. i_{PH} represents the current generated by absorption of photons in the vicinity of the *pn* junction.

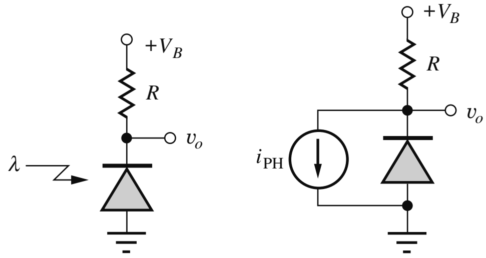


Figure 3.71 Basic photodetector circuit (a) and model (b).

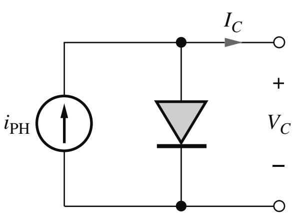


Figure 3.72 *pn* Diode under steady-state illumination as a solar cell.

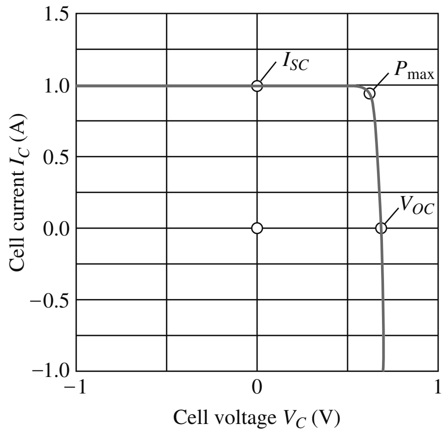


Figure 3.73 Terminal characteristics for a *pn* junction solar cell.

In optical fiber communication systems, the amplitude of the incident light is modulated by rapidly changing digital data, and i_{PH} includes a time-varying signal component. The time-varying signal voltage at v_o is fed to additional electronic circuits to demodulate the signal and recover the original data that were transmitted down the optical fiber.

3.18.2 POWER GENERATION FROM SOLAR CELLS

In **solar cell** applications, the optical illumination is constant, and a dc current I_{PH} is generated. The goal is to extract power from the cell, and the *i-v* characteristics of solar cells are usually plotted in terms of the cell current I_C and cell voltage V_C, as defined in Fig. 3.72.

The *i-v* characteristic of the *pn* junction used for solar cell applications is plotted in terms of these terminal variables in Fig. 3.73. Also indicated on the graph are the short-circuit current I_{SC}, the open-circuit voltage V_{OC}, and the maximum power point P_{max}. I_{SC} represents the maximum current available from the cell, and V_{OC} is the voltage across the open-circuited cell when all the photo current is flowing into the internal *pn* junction. For the solar cell to supply power to an external circuit, the product $I_C \times V_C$ must be positive, corresponding to the first quadrant of the characteristic. An attempt is made to operate the cell near the point of maximum output power P_{max}.

ELECTRONICS IN ACTION

Solar Energy

The photograph below depicts the Long Island Solar Farm installation on the Brookhaven National Laboratory (BNL) site in the center of Long Island, New York. The installation, consisting of 164,312 solar panels utilizing crystalline silicon technology, is capable of a generating a peak power of 32 MW with an estimated annual energy output of 44 million kilowatt-hours, enough to power an estimated 4500 homes for a year. The project was a collaboration between the Department of Energy, BP Solar, and the Long Island Power Authority and became operational near the end of 2011. The Long Island Power Authority purchases 100 percent of the power generated by the installation that is estimated to offset production of approximately 30,000 metric tons of CO_2 per year as well as significant amounts of other pollutants.

Long Island Solar Farm Installation. The main BNL campus is at the upper center of the picture. *Courtesy Brookhaven National Laboratory*.

3.18.3 LIGHT-EMITTING DIODES (LEDs)

Light-emitting diodes, or **LEDs,** rely on the annihilation of electrons and holes through recombination rather than on the generation of carriers, as in the case of the photo diode. When a hole and electron recombine, an energy equal to the bandgap of the semiconductor can be released in the form of a photon. This recombination process is present in the forward-biased *pn* junction diode. In silicon, the recombination process actually involves the interaction of photons and lattice vibrations called phonons, so the optical emission process in silicon is not nearly as efficient as that in the III–V compound semiconductor GaAs or the ternary materials such as $GaIn_{1-x}As_x$ and $GaIn_{1-x}P_x$. LEDs in these compound semiconductor materials provide visible illumination, and the color of the output can be controlled by varying the fraction *x* of arsenic or phosphorus in the material which changes this bandgap energy.

SUMMARY

In this chapter we investigated the detailed behavior of the solid-state diode.

- A *pn* junction diode is created when *p*-type and *n*-type semiconductor regions are formed in intimate contact with each other. In the *pn* diode, large concentration gradients exist in the vicinity of the metallurgical junction, giving rise to large electron and hole diffusion currents.
- Under zero bias, no current can exist at the diode terminals, and a space charge region forms in the vicinity of the *pn* junction. The region of space charge results in both a built-in potential and an internal electric field, and the electric field produces electron and hole drift currents that exactly cancel the corresponding components of diffusion current.
- When a voltage is applied to the diode, the balance in the junction region is disturbed, and the diode conducts a current. The resulting i-v characteristics of the diode are accurately modeled by the diode equation:

$$i_D = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right]$$

  where I_S = reverse saturation current of the diode

  n = nonideality factor (approximately 1)

  $V_T = kT/q$ = thermal voltage (0.025 V at room temperature)

- Under reverse bias, the diode current equals $-I_S$, a very small current.
- For forward bias, however, large currents are possible, and the diode presents an almost constant voltage drop of 0.6 to 0.7 V.
- At room temperature, an order of magnitude change in diode current requires a change of less than 60 mV in the diode voltage. At room temperature, the silicon diode voltage exhibits a temperature coefficient of approximately -1.8 mV/°C.
- One must also be aware of the reverse-breakdown phenomenon that is not included in the diode equation. If too large a reverse voltage is applied to the diode, the internal electric field becomes so large that the diode enters the breakdown region, either through Zener breakdown or avalanche breakdown. In the breakdown region, the diode again represents an almost fixed voltage drop, and the current must be limited by the external circuit or the diode can easily be destroyed.
- Diodes called Zener diodes are designed to operate in breakdown and can be used in simple voltage regulator circuits. Line regulation and load regulation characterize the change in output voltage of a power supply due to changes in input voltage and output current, respectively.
- As the voltage across the diode changes, the charge stored in the vicinity of the space charge region of the diode changes, and a complete diode model must include a capacitance. Under reverse bias, the capacitance varies inversely with the square root of the applied voltage. Under forward bias, the capacitance is proportional to the operating current and the diode transit time. These capacitances prevent the diode from turning on and off instantaneously and cause a storage time delay during turn-off.
- Direct use of the nonlinear diode equation in circuit calculations usually requires iterative numeric techniques. Several methods for simplifying the analysis of diode circuits were discussed, including the graphical load-line method and use of the ideal diode and constant voltage drop models.
- SPICE circuit analysis programs include a comprehensive built-in model for the diode that accurately reproduces both the ideal and nonideal characteristics of the diode and is useful for exploring the detailed behavior of circuits containing diodes.

- Important applications of diodes include half-wave, full-wave, and full-wave bridge rectifier circuits used to convert from ac to dc voltages in power supplies. Simple power supply circuits use capacitive filters, and the design of the filter capacitor determines power supply ripple voltage and diode conduction angle. Diodes used as rectifiers in power supplies must be able to withstand large peak repetitive currents as well as surge currents when the power supplies are first turned on. The reverse-breakdown voltage of rectifier diodes is referred to as the peak-inverse-voltage, or PIV, rating of the diode.
- Real diodes cannot turn on or off instantaneously because the internal capacitances of the diodes must be charged and discharged. The turn-on time is usually quite short, but diodes that have been conducting turn off much less abruptly. It takes time to remove stored charge within the diode, and this time delay is characterized by storage time τ_s. During the storage time, it is possible for large reverse currents to occur in the diode.
- Finally, the ability of the *pn* junction device to generate and detect light was discussed, and the basic characteristics of photo diodes, solar cells, and light-emitting diodes were presented.

KEY TERMS

Anode
Avalanche breakdown
Bias current and voltage
Breakdown region
Breakdown voltage
Built-in potential (or voltage)
Cathode
Center-tapped transformer
Conduction angle
Conduction interval
Constant voltage drop (CVD) model
Cut-in voltage
Depletion layer
Depletion-layer capacitance
Depletion-layer width
Depletion region
Diffusion capacitance
Diode equation
Diode SPICE parameters (IS, RS, N, TT, CJO, VJ, M)
Filter capacitor
Forward bias
Full-wave bridge rectifier circuit
Full-wave rectifier circuit
Half-wave rectifier circuit
Ideal diode
Ideal diode model
Impact-ionization process
Junction capacitance
Junction potential
Light-emitting diode (LED)
Line regulation
Load line
Load-line analysis
Load regulation
Mathematical model
Metallurgical junction
Nonideality factor (n)
Peak detector
Peak inverse voltage (PIV)
Photodetector circuit
Piecewise linear model
pn junction diode
Q-point
Quiescent operating point
Rectifier circuits
Reverse bias
Reverse breakdown
Reverse saturation current (I_S)
Ripple current
Ripple voltage
Saturation current
Schottky barrier diode
Solar cell
Space charge region (SCR)
Storage time
Surge current
Thermal voltage (V_T)
Transit time
Turn-on voltage
Voltage regulator
Voltage transfer characteristic (VTC)
Zener breakdown
Zener diode
Zero bias
Zero-bias junction capacitance

REFERENCE

1. G. W. Neudeck, *The PN Junction Diode,* 2d ed. Pearson Education, Upper Saddle River, NJ: 1989.

ADDITIONAL READING

PSPICE, ORCAD, now owned by Cadence Design Systems, San Jose, CA.

LTspice available from Linear Technology Corp.

Tina-TI SPICE-based analog simulation program available from Texas Instruments.

T. Quarles, A. R. Newton, D. O. Pederson, and A. Sangiovanni-Vincentelli, *SPICE3 Version 3f3 User's Manual.* UC Berkeley: May 1993.

A. S. Sedra, and K. C. Smith. *Microelectronic Circuits.* 5th ed. Oxford University Press, New York: 2004.

PROBLEMS

3.1 The *pn* Junction Diode

3.1. A diode is doped with $N_A = 10^{18}/\text{cm}^3$ on the p-type side and $N_D = 10^{19}/\text{cm}^3$ on the n-type side. (a) What is the depletion-layer width w_{do}? (b) What are the values of x_p and x_n? (c) What is the value of the built-in potential of the junction? (d) What is the value of E_{MAX}? Use Eq. (3.3) and Fig. 3.5.

3.2. A diode is doped with $N_A = 10^{18}/\text{cm}^3$ on the p-type side and $N_D = 10^{18}/\text{cm}^3$ on the n-type side. (a) What are the values of p_p, p_n, n_p, and n_n? (b) What are the depletion-region width w_{do} and built-in voltage?

3.3. Repeat Prob. 3.2 for a diode with $N_A = 10^{16}/\text{cm}^3$ on the p-type side and $N_D = 10^{20}/\text{cm}^3$ on the n-type side.

3.4. Repeat Prob. 3.2 for a diode with $N_A = 10^{18}/\text{cm}^3$ on the p-type side and $N_D = 10^{18}/\text{cm}^3$ on the n-type side.

3.5. A diode has $w_{do} = 1$ μm and $\phi_j = 0.7$ V. (a) What reverse bias is required to double the depletion-layer width? (b) What is the depletion region width if a reverse bias of 12 V is applied to the diode?

3.6. A diode has $w_{do} = 0.4$ μm and $\phi_j = 0.85$ V. (a) What reverse bias is required to triple the depletion-layer width? (b) What is the depletion region width if a reverse bias of 7 V is applied to the diode?

3.7. Suppose a drift current density of 5000 A/cm^2 exists in the neutral region on the n-type side of a diode that has a resistivity of 0.5 Ω · cm. What is the electric field needed to support this drift current density?

3.8. Suppose a drift current density of 2000 A/cm^2 exists in the neutral region on the p-type side of a diode that has a resistivity of 2.5 Ω · cm. What is the electric field needed to support this drift current density?

3.9. The maximum velocity of carriers in silicon is approximately 10^7 cm/s. (a) What is the maximum drift current density that can be supported in a region of p-type silicon with a doping of $5 \times 10^{17}/\text{cm}^3$? (b) Repeat for a region of n-type silicon with a doping of $4 \times 10^{15}/\text{cm}^3$?

**3.10. Suppose that $N_A(x) = N_o \exp(-x/L)$ in a region of silicon extending from $x = 0$ to $x = 8$ μm, where N_o is a constant. Assume that $p(x) = N_A(x)$. Assuming that j_p must be zero in thermal equilibrium, show that a built-in electric field must exist and find its value for $L = 1$ μm and $N_o = 10^{18}/\text{cm}^3$.

3.11. What carrier gradient is needed to generate a diffusion current density of $j_n = 1500$ A/cm^2 if $\mu_n = 500$ cm^2/V · s?

3.12. Use the solver routine in your calculator to find the solution to Eq. (3.25) for $I_S = 10^{-16}$ A.

3.13. Use a spreadsheet to iteratively find the solution to Eq. (3.25) for $I_S = 10^{-13}$ A.

3.14. (a) Use MATLAB or MATHCAD to find the solution to Eq. (3.25) for $I_S = 10^{-13}$ A. (b) Repeat for $I_S = 10^{-15}$ A.

3.2 –3.4 The *i-v* Characteristics of the Diode; the Diode Equation: a Mathematical Model for the Diode; and Diode Characteristics Under Reverse, Zero, and Forward Bias

3.15. To what temperature does $V_T = 0.025$ V actually correspond? What is the value of V_T for temperatures of $-55°$C, $0°$C, and $+85°$C?

3.16. (a) Plot a graph of the diode equation similar to Fig. 3.8 for a diode with $I_S = 10^{-12}$ A and $n = 1$. (b) Repeat for $n = 2$. (c) Repeat (a) for $I_S = 10^{-15}$ A.

3.17. A diode has $n = 1.06$ at $T = 320$ K. What is the value of $n \cdot V_T$? What temperature would give the same value of $n \cdot V_T$ if $n = 1.00$?

3.18. Plot the diode current for a diode with $I_{SO} = 15$ fA and $\phi_j = 0.75$ V for $-10 \text{ V} \le v_D \le 0$ V using Eq. (3.19).

*3.19. What are the values of I_S and n for the diode in the graph in Fig. P3.19? Assume $V_T = 0.0259$ V.

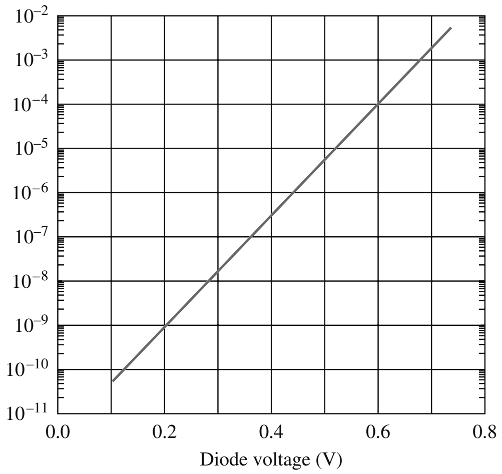


Figure P3.19

3.20. A diode has $I_S = 10^{-17}$ A and $n = 1.05$. (a) What is the diode voltage if the diode current is 70 μA? (b) What is the diode voltage if the diode current is 5 μA? (c) What is the diode current for $v_D = 0$ V? (d) What is the diode current for $v_D = -0.075$ V? (e) What is the diode current for $v_D = -5$ V?

3.21. A diode has $I_S = 5$ aA and $n = 1$. (a) What is the diode voltage if the diode current is 100 μA? (b) What is the diode voltage if the diode current is 10 μA? (c) What is the diode current for $v_D = 0$ V? (d) What is the diode current for $v_D = -0.06$ V? (e) What is the diode current for $v_D = -4$ V?

3.22. A diode has $I_S = 0.2$ fA and $n = 1$. (a) What is the diode current if the diode voltage is 0.675 V? (b) What will be the diode voltage if the current increases by a factor of 3?

3.23. A diode has $I_S = 10^{-10}$ A and $n = 2$. (a) What is the diode voltage if the diode current is 40 A? (b) What is the diode voltage if the diode current is 100 A?

3.24. A diode is operating with $i_D = 2$ mA and $v_D = 0.82$ V. (a) What is I_S if $n = 1$? (b) What is the diode current for $v_D = -5$ V?

3.25. A diode is operating with $i_D = 300$ μA and $v_D = 0.75$ V. (a) What is I_S if $n = 1.07$? (b) What is the diode current for $v_D = -3$ V?

3.26. The saturation current for diodes with the same part number may vary widely. Suppose it is known that $10^{-14} \text{ A} \le I_S \le 10^{-12}$ A. What is the range of forward voltages that may be exhibited by the diode if it is biased with $i_D = 2$ mA?

3.27. A diode is biased by a 0.9-V dc source, and its current is found to be 100 μA at $T = 35\,°$C. (a) At what temperature will the current double? (b) At what temperature will the current be 50 μA?

**3.28. The i-v characteristic for a diode has been measured under carefully controlled temperature conditions ($T = 307$ K), and the data are in Table P3.28.

TABLE P3.28
Diode i-v Measurements

DIODE VOLTAGE	DIODE CURRENT
0.500	6.591×10^{-7}
0.550	3.647×10^{-6}
0.600	2.158×10^{-5}
0.650	1.780×10^{-4}
0.675	3.601×10^{-4}
0.700	8.963×10^{-4}
0.725	2.335×10^{-3}
0.750	6.035×10^{-3}
0.775	1.316×10^{-2}

Use a spreadsheet or MATLAB to find the values of I_S and n that provide the best fit of the diode equation to the measurements in the least-squares sense. [That is, find the values of I_S and n that minimize the function $M = \sum_{m=1}^{n}(i_D^m - I_{Dm})^2$, where i_D is the diode equation from Eq. (3.1) and I_{Dm} are the measured data.] For your values of I_S and n, what is the minimum value of $M = \sum_{m=1}^{n}(i_D^m - I_{Dm})^2$?

3.5 Diode Temperature Coefficient

3.29. What is the value of V_T for temperatures of $-40°$C, $0°$C, and $+50°$C?

3.30. A diode has $I_S = 10^{-16}$ A and $n = 1$. (a) What is the diode voltage if the diode current is 250 μA at $T = 25°C$? (b) What is the diode voltage at $T = 85°C$? Assume the diode voltage temperature coefficient is -2 mV/K at 55°C.

3.31. A diode has $I_S = 20$ fA and $n = 1$. (a) What is the diode voltage if the diode current is 100 μA at $T = 25°C$? (b) What is the diode voltage at $T = 50°C$? Assume the diode voltage temperature coefficient is -1.8 mV/K at 0°C.

*3.32. The temperature dependence of I_S is described approximately by

$$I_S = CT^3 \exp\left(-\frac{E_G}{kT}\right)$$

What is the diode voltage temperature coefficient based on this expression and Eq. (3.15) if $E_G = 1.12$ eV, $V_D = 0.7$ V, and $T = 315$ K?

3.33. The saturation current of a silicon diode is described by the expression in Prob. 3.32. (a) What temperature change will cause I_S to double? (b) To increase by 10 times? (c) To decrease by 100 times?

3.6 Diodes under Reverse Bias

3.34. A diode has $w_{do} = 1.5$ μm and $\phi_j = 0.8$ V. (a) What is the depletion layer width for $V_R = 5$ V? (b) For $V_D = -10$ V?

3.35. A diode has a doping of $N_D = 10^{15}/\text{cm}^3$ on the n-type side and $N_A = 10^{17}/\text{cm}^3$ on the p-type side. What are the values of w_{do} and ϕ_j? What is the value of w_d at a reverse bias of 10 V? At 100 V?

3.36. A diode has a doping of $N_D = 10^{20}/\text{cm}^3$ on the n-type side and $N_A = 10^{18}/\text{cm}^3$ on the p-type side. What are the values of w_{do} and ϕ_j? What is the value of w_d at a reverse bias of 5 V? At 25 V?

*3.37. A diode has $w_{do} = 2$ μm and $\phi_j = 0.6$ V. If the diode breaks down when the internal electric field reaches 300 kV/cm, what is the breakdown voltage of the diode?

*3.38. Silicon breaks down when the internal electric field exceeds 300 kV/cm. At what reverse bias do you expect the diode of Prob. 3.2 to break down?

3.39. What are the breakdown voltage V_Z and Zener resistance R_Z of the diode depicted in Fig. P3.39?

**3.40. A diode is fabricated with $N_A \gg N_D$. What value of doping is required on the lightly doped side to achieve a reverse-breakdown voltage of 750 V if the semiconductor material breaks down at a field of 300 kV/cm?

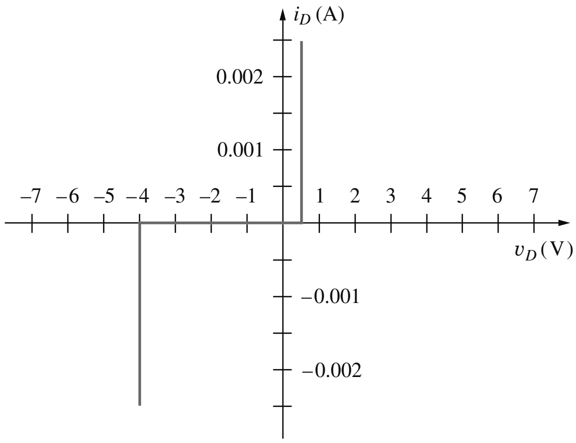


Figure P3.39

3.7 *pn* Junction Capacitance

3.41. What is the zero-bias junction capacitance/cm² for a diode with $N_A = 10^{18}/\text{cm}^3$ on the p-type side and $N_D = 10^{20}/\text{cm}^3$ on the n-type side? What is the diode capacitance with a 3-V reverse bias if the diode area is 0.05 cm²?

3.42. What is the zero-bias junction capacitance per cm² for a diode with $N_A = 10^{18}/\text{cm}^3$ on the p-type side and $N_D = 10^{15}/\text{cm}^3$ on the n-type side. What is the diode capacitance with a 9 V reverse bias if the diode area is 0.02 cm²?

3.43. A diode is operating at a current of 250 μA. (a) What is the diffusion capacitance if the diode transit time is 100 ps? (b) How much charge is stored in the diode? (c) Repeat for $i_D = 3$ mA.

3.44. A diode is operating at a current of 2 A. (a) What is the diffusion capacitance if the diode transit time is 10 ns? (b) How much charge is stored in the diode? (c) Repeat for $i_D = 100$ mA.

3.45. A square pn junction diode is 5 mm on a side. The p-type side has a doping concentration of $10^{19}/\text{cm}^3$ and the n-type side has a doping concentration of $10^{16}/\text{cm}^3$. (a) What is the zero-bias capacitance of the diode? What is the capacitance at a reverse bias of 4 V? (b) Repeat for an area of 10^4 μm².

3.46. A variable capacitance diode with $C_{jo} = 39$ pF and $\phi_j = 0.80$ V is used to tune a resonant LC circuit as shown in Fig. P3.46. The impedance of the RFC (radio frequency choke) can be considered infinite. What are the resonant frequencies ($f_o = \frac{1}{2\pi\sqrt{LC}}$) for $V_{DC} = 1$ V and $V_{DC} = 9$ V?

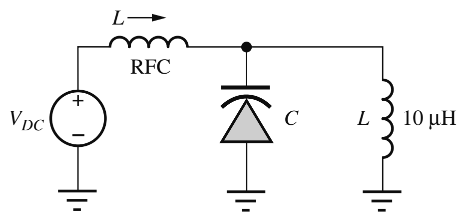


Figure P3.46

3.8 Schottky Barrier Diode

3.47. A Schottky barrier diode is modeled by the diode equation in Eq. (3.11) with $I_S = 10^{-11}$ A. (a) What is the diode voltage at a current of 4 mA? (b) What would be the voltage of a *pn* junction diode with $I_S = 10^{-14}$ A operating at the same current?

3.48. Suppose a Schottky barrier diode can be modeled by the diode equation in Eq. (3.11) with $I_S = 10^{-7}$ A. (a) What is the diode voltage at a current of 50 A? (b) What would be the voltage of a *pn* junction diode with $I_S = 10^{-15}$ A and $n = 2$?

3.9 Diode SPICE Model and Layout

3.49. (a) A diode has $I_S = 5 \times 10^{-16}$ A and $R_S = 10\ \Omega$ and is operating at a current of 1 mA at room temperature. What are the values of V_D and V_D'? (b) Repeat for $R_S = 100\ \Omega$.

3.50. A *pn* diode has a resistivity of 1 $\Omega \cdot$ cm on the *p*-type side and 0.02 $\Omega \cdot$ cm on the *n*-type side. What is the value of R_S for this diode if the cross-sectional area of the diode is 0.01 cm^2 and the lengths of the *p*- and *n*-sides of the diode are each 250 µm?

*3.51. A diode fabrication process has a specific contact resistance of 10 $\Omega \cdot \mu m^2$. If the contacts are each 1 µm × 1 µm in size, what are the total contact resistances associated with the anode and cathode contacts to the diode in Fig. 3.21(a).

3.10 Diode Circuit Analysis

3.52. (a) Plot the load line and find the Q-point for the diode circuit in Fig. P3.53 if $V = 5$ V and $R = 10\ k\Omega$. Use the *i*-*v* characteristic in Fig. P3.39. (b) Repeat for $V = -6$ V and $R = 3\ k\Omega$. (c) Repeat for $V = -3$ V and $R = 3\ k\Omega$.

3.53. (a) Plot the load line and find the Q-point for the diode circuit in Fig. P3.53 if $V = 10$ V and $R = 5\ k\Omega$. Use the *i*-*v* characteristic in Fig. P3.39. (b) Repeat for $V = -10$ V and $R = 5\ k\Omega$. (c) Repeat for $V = -2$ V and $R = 2\ k\Omega$.

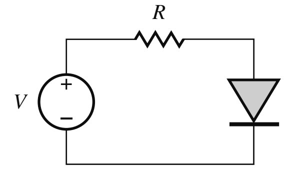


Figure P3.53

3.54. Simulate the circuit in Prob. 3.53 with SPICE and compare the results to those in Prob. 3.53. Use $I_S = 10^{-15}$ A.

3.55. Use the *i*-*v* characteristic in Fig. P3.39. (a) Plot the load line and find the Q-point for the diode circuit in Fig. P3.53 if $V = 6$ V and $R = 4\ k\Omega$. (b) For $V = -6$ V and $R = 3\ k\Omega$. (c) For $V = -3$ V and $R = 3\ k\Omega$. (d) For $V = 12$ V and $R = 8\ k\Omega$. (e) For $V = -25$ V and $R = 10\ k\Omega$.

Iterative Analysis and the Mathematical Model

3.56. Use direct trial and error to find the solution to the diode circuit in Fig. 3.22 using Eq. (3.27).

3.57. Repeat the iterative procedure used in the spreadsheet in Table 3.2 for initial guesses of 1 µA, 5 mA, and 5 A and 0 A. How many iterations are required for each case? Did any problem arise? If so, what is the source of the problem?

3.58. A diode has $I_S = 0.1$ fA and is operating at $T = 300$ K. (a) What are the values of V_{DO} and r_D if $I_D = 200\ \mu A$? (b) If $I_D = 2.0$ mA? (c) If $I_D = 20$ mA?

3.59. (a) Use the iterative procedure in the spreadsheet in Table 3.2 to find the diode current and voltage for the circuit in Fig. 3.22 if $V = 2.5$ V and $R = 3\ k\Omega$. (b) Repeat for $V = 7.5$ V and $R = 15\ k\Omega$.

3.60. (a) Use the iterative procedure in the spreadsheet in Table 3.2 to find the diode current and voltage for the circuit in Fig. 3.22 if $V = 1$ V and $R = 15\ k\Omega$. (b) Repeat for $V = 3$ V and $R = 6.2\ k\Omega$.

3.61. Use MATLAB or MATHCAD to numerically find the Q-point for the circuit in Fig. 3.22 using the equation in the exercise on page 99.

Ideal Diode and Constant Voltage Drop Models

*3.62. Find the Q-point for the circuit in Fig. 3.22 using the same four methods as in Sec. 3.10 if the voltage source is 1 V. Compare the answers in a manner similar to Table 3.3.

3.63. Find the Q-point for the diode in Fig. P3.63 using (a) the ideal diode model and (b) the constant voltage drop model with $V_{on} = 0.6$ V. (c) Discuss the results. Which answer do you feel is most correct? (d) Use iterative analysis to find the actual Q-point if $I_S = 0.1$ fA.

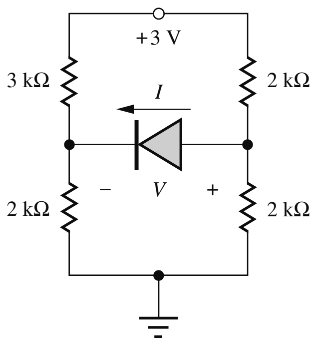


Figure P3.63

3.64. Simulate the circuit of Fig. P3.63 and find the diode Q-point. Compare the results to those in Prob. 3.63.

3.65. (a) Find the worst-case values of the Q-point current for the diode in Fig. P3.63 using the ideal diode model if the resistors all have 10 percent tolerances. (b) Repeat using the CVD model with $V_{on} = 0.6$ V.

3.66. (a) Find I and V in the four circuits in Fig. P3.66 using the ideal diode model. (b) Repeat using the constant voltage drop model with $V_{on} = 0.65$ V.

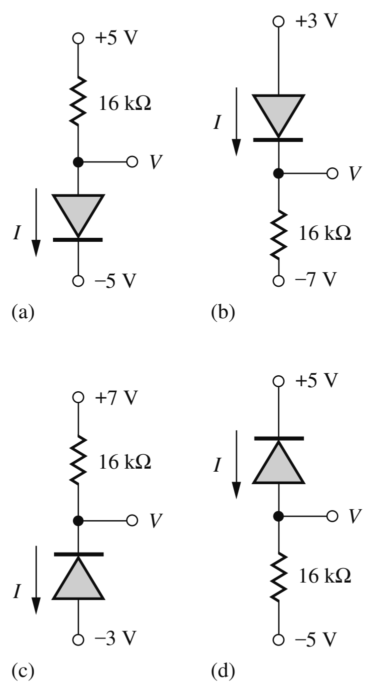


Figure P3.66

3.67. (a) Find I and V in the four circuits in Fig. P3.66 using the ideal diode model if the resistor values are changed to 68 kΩ. (b) Repeat using the constant voltage drop model with $V_{on} = 0.6$ V.

3.11 Multiple Diode Circuits

3.68. Find the Q-points for the diodes in the four circuits in Fig. P3.68 using (a) the ideal diode model and (b) the constant voltage drop model with $V_{on} = 0.7$ V.

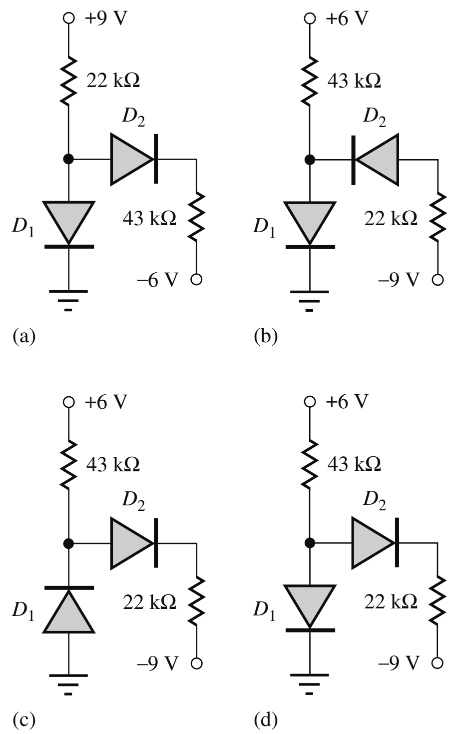


Figure P3.68

3.69. Find the Q-points for the diodes in the four circuits in Fig. P3.68 if the values of all the resistors are changed to 15 kΩ using (a) the ideal diode model and (b) the constant voltage drop model with $V_{on} = 0.60$ V.

3.70. Find the Q-point for the diodes in the circuits in Fig. P3.70 using the ideal diode model.

3.71. Find the Q-point for the diodes in the circuits in Fig. P3.70 using the constant voltage drop model with $V_{on} = 0.65$ V.

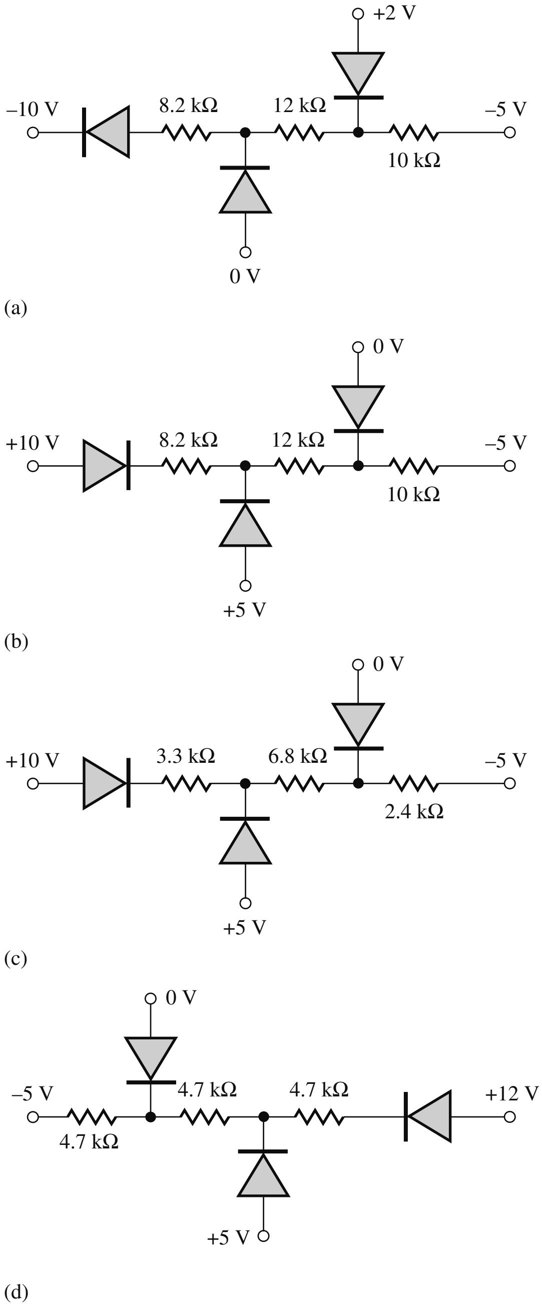


Figure P3.70

3.72. Simulate the diode circuits in Fig. P3.70 and compare your results to those in Prob. 3.70.

3.73. Verify that the values presented in Ex. 3.8 using the ideal diode model are correct.

3.74. Simulate the circuit in Fig. 3.33 and compare to the results in Ex. 3.8.

3.12 Analysis of Diodes Operating in the Breakdown Region

3.75. Draw the load line for the circuit in Fig. P3.75 on the characteristics in Fig. P3.39 and find the Q-point.

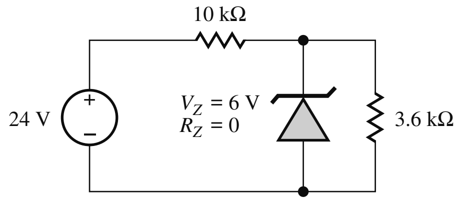


Figure P3.75

3.76. (a) Find the Q-point for the Zener diode in Fig. P3.75. (b) Repeat if $R_Z = 100\ \Omega$.

3.77. What is maximum load current I_L that can be drawn from the Zener regulator in Fig. P3.77 if it is to maintain a regulated output? What is the minimum value of R_L that can be used and still have a regulated output voltage?

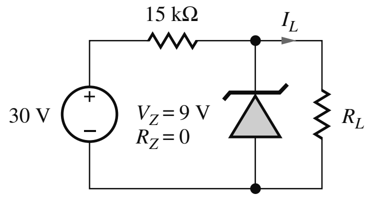


Figure P3.77

3.78. What is power dissipation in the Zener diode in Fig. P3.77 for (a) $R_L = 2\ \text{k}\Omega$ (b) $R_L = 4.7\ \text{k}\Omega$ (c) $R_L = 15\ \text{k}\Omega$ (d) $R_L = \infty$?

3.79. Load resistor R_L in Fig. P3.77 is 12 kΩ. What are the nominal and worst-case values of Zener diode current and power dissipation if the power supply voltage, Zener breakdown voltage and resistors all have 5 percent tolerances?

3.80. What is power dissipation in the Zener diode in Fig. P3.80 for (a) $R_L = 100\ \Omega$? (b) $R_L = \infty$?

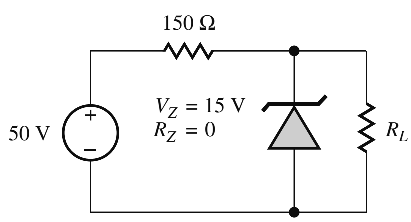


Figure P3.80

3.81. Load resistor R_L in Fig. P3.80 is 100 Ω. What are the nominal and worst-case values of Zener diode current and power dissipation if the power supply voltage, Zener breakdown voltage, and resistors all have 10 percent tolerances?

3.13 Half-Wave Rectifier Circuits

3.82. A power diode has a reverse saturation current of 10^{-9} A and $n = 1.6$. What is the forward voltage drop at the peak current of 48.6 A that was calculated in the example in Sec. 3.13.5?

3.83. A power diode has a reverse saturation current of 10^{-8} A and $n = 2$. What is the forward voltage drop at the peak current of 100 A? What is the power dissipation in the diode in a half-wave rectifier application operating at 60 Hz if the series resistance is 0.01 Ω and the conduction time is 1 ms?

*3.84. (a) Use a spreadsheet or MATLAB or write a computer program to find the numeric solution to the conduction angle equation for a 60 Hz half-wave rectifier circuit that uses a filter capacitance of 100,000 µF. The circuit is designed to provide 5 V at 5 A. {That is, solve $[(V_P - V_{on})\exp(-t/RC) = V_P \cos \omega t - V_{on}]$. Be careful! There are an infinite number of solutions to this equation. Be sure your algorithm finds the desired answer to the problem.} Assume $V_{on} = 1$ V. (b) Compare to calculations using Eq. (3.57).

3.85. What is the actual average value (the dc value) of the rectifier output voltage for the waveform in Fig. P3.85 if V_r is 10 percent of $V_P - V_{on} = 18$ V?

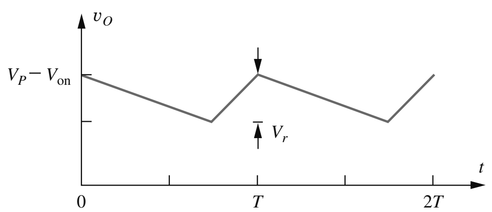


Figure P3.85

3.86. Draw the voltage waveforms, similar to those in Fig. 3.53, for the negative output rectifier in Fig. 3.57(b).

*3.87. Show that evaluation of Eq. (3.61) will yield the result in Eq. (3.62).

3.88. The half-wave rectifier in Fig. P3.88 is operating at a frequency of 60 Hz, and the rms value of the transformer output voltage v_I is 12.6 V ± 10%. What are the nominal and worst case values of the dc output voltage V_O if the diode voltage drop is 1 V?

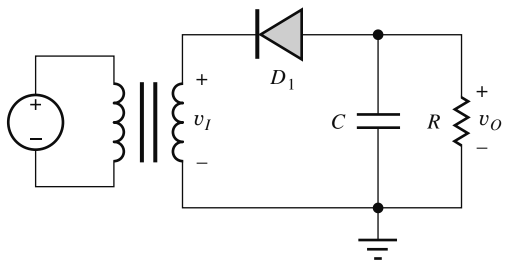


Figure P3.88

3.89. The half-wave rectifier in Fig. P3.88 is operating at a frequency of 60 Hz, and the rms value of the transformer output voltage is 6.3 V. (a) What is the value of the dc output voltage V_O if the diode voltage drop is 1 V? (b) What is the minimum value of C required to maintain the ripple voltage to less than 0.25 V if $R = 0.5$ Ω? (c) What is the PIV rating of the diode in this circuit? (d) What is the surge current when power is first applied? (e) What is the amplitude of the repetitive current in the diode?

3.90. Simulate the behavior of the half-wave rectifier in Fig. P3.88 for $v_I = 10 \sin 120\pi t$, $R = 0.025$ Ω and $C = 0.5$ F. (Use IS $= 10^{-10}$ A, RS $= 0$, and RELTOL $= 10^{-6}$.) Compare the simulated values of dc output voltage, ripple voltage, and peak diode current to hand calculations. Repeat simulation with $R_S = 0.02$ Ω.

3.91. (a) Repeat Prob. 3.89 for a frequency of 400 Hz. (b) Repeat Prob. 3.89 for a frequency of 70 kHz.

3.92. A 3.3-V, 30-A dc power supply is to be designed with a ripple of less than 1.5 percent. Assume that a half-wave rectifier circuit (60 Hz) with a capacitor filter is used. (a) What is the size of the filter capacitor C? (b) What is the PIV rating for the diode? (c) What is the rms value of the transformer voltage needed for the rectifier? (d) What is the value of the peak repetitive diode current in the diode? (e) What is the surge current at $t = 0^+$?

3.93. A 2500-V, 2-A, dc power supply is to be designed with a ripple voltage ≤ 0.5 percent. Assume that a half-wave rectifier circuit (60 Hz) with a capacitor filter is used. (a) What is the size of the filter capacitor C? (b) What is the minimum PIV rating for the diode? (c) What is the rms value of the transformer voltage needed for the rectifier? (d) What is the peak value of the repetitive current in the diode? (e) What is the surge current at $t = 0^+$?

*3.94. Draw the voltage waveforms at nodes v_O and v_1 for the "voltage-doubler" circuit in Fig. P3.94 for the first two cycles of the input sine wave. What is the steady-state output voltage if $V_P = 17$ V?

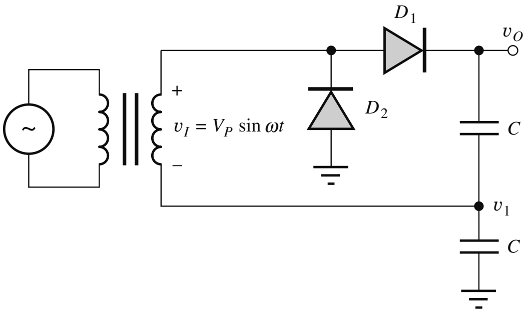


Figure P3.94

3.95. Simulate the voltage-doubler rectifier circuit in Fig. P3.94 for $C = 500$ μF and $v_I = 1500 \sin 2\pi(60)t$ with a load resistance of $R_L = 3000\ \Omega$ added between v_O and ground. Calculate the ripple voltage and compare to the simulation.

3.96. Estimate the maximum surge current in a half-wave rectifier with a transformer having an rms secondary voltage of 50 V and a secondary resistance of 0.25 Ω. Assume the filter capacitance is 0.5 F and a frequency of 60 Hz.

3.14 Full-Wave Rectifier Circuits

3.97. The full-wave rectifier in Fig. P3.97 is operating at a frequency of 60 Hz, and the rms value of the transformer output voltage is 18 V. (a) What is the value of the dc output voltage if the diode voltage drop is 1 V? (b) What is the minimum value of C required to maintain the ripple voltage to less than 0.25 V if $R = 0.5\ \Omega$? (c) What is the PIV rating of the diode in this circuit? (d) What is the surge current when power is first applied? (e) What is the amplitude of the repetitive current in the diode?

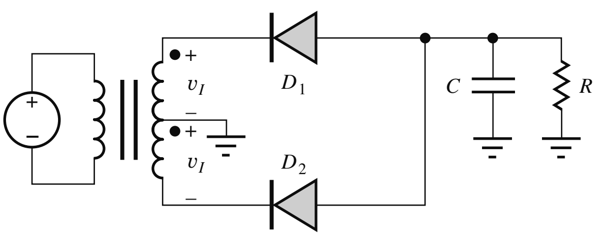


Figure P3.97

3.98. Repeat Prob. 3.97 if the rms value of the transformer output voltage v_I is 15 V.

3.99. A 60-Hz full-wave rectifier is built with a transformer having an rms secondary voltage of 20 V and filter capacitance $C = 150{,}000$ μF. What is the largest current that can be supplied by the rectifier circuit if the ripple must be less than 0.3 V?

3.100. Simulate the behavior of the full-wave rectifier in Fig. P3.97 for $R = 3\ \Omega$ and $C = 22{,}000$ μF. Assume that the rms value of v_I is 10.0 V and the frequency is 400 Hz. (Use IS $= 10^{-10}$ A, RS $= 0$, and RELTOL $= 10^{-6}$.) Compare the simulated values of dc output voltage, ripple voltage, and peak diode current to hand calculations. Repeat simulation with $R_S = 0.25$.

3.101. Repeat Prob. 3.92 for a full-wave rectifier circuit.

3.102. Repeat Prob. 3.93 for a full-wave rectifier circuit.

*3.103. The full-wave rectifier circuit in Fig. P3.103(a) was designed to have a maximum ripple of approximately 1 V, but it is not operating properly. The measured waveforms at the three nodes in the circuit are shown in Fig. P3.103(b). What is wrong with the circuit?

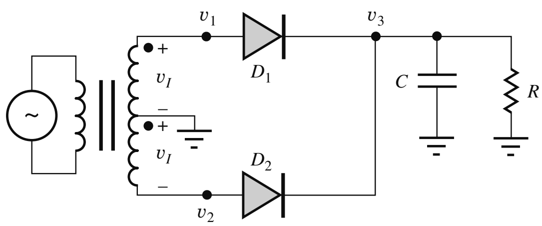


Figure P3.103(a)

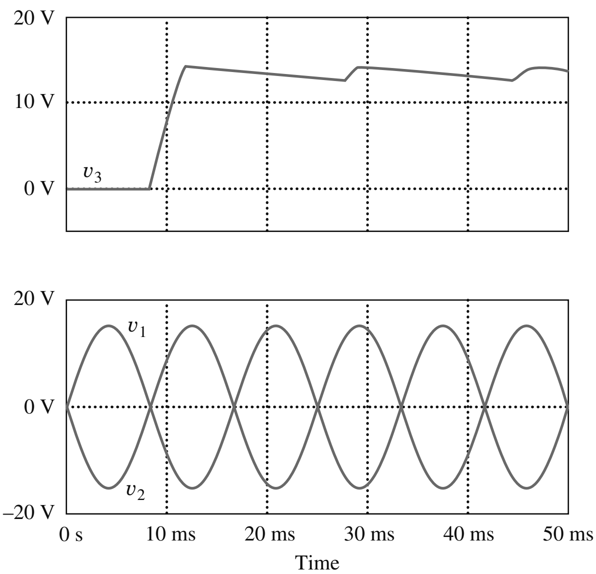


Figure P3.103(b) Waveforms for the circuit in Fig. P3.103(a).

3.104. For the Zener regulated power supply in Fig. P3.104, the rms value of v_I is 15 V, the operating frequency is 60 Hz, $R = 100\ \Omega$, $C = 1000\ \mu F$, the on-voltage of diodes D_1 and D_2 is 0.75 V, and the Zener voltage of diode D_3 is 15 V. (a) What type of rectifier is used in this power supply circuit? (b) What is the dc voltage at V_1? (c) What is the dc output voltage V_O? (d) What is the magnitude of the ripple voltage at V_1? (e) What is the minimum PIV rating for the rectifier diodes? (f) Draw a new version of the circuit that will produce an output voltage of -15 V.

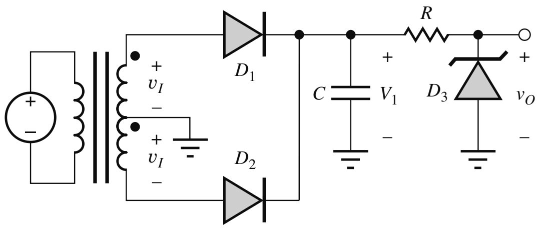


Figure P3.104

3.15 Full-Wave Bridge Rectification

3.105. Repeat Prob. 3.97 for a full-wave bridge rectifier circuit. Draw the circuit.

3.106. Repeat Prob. 3.92 for a full-wave bridge rectifier circuit. Draw the circuit.

3.107. Repeat Prob. 3.93 for a full-wave bridge rectifier circuit. Draw the circuit.

*3.108. What are the dc output voltages V_1 and V_2 for the rectifier circuit in Fig. P3.108 if $v_I = 40 \sin 377t$ and $C = 20{,}000\ \mu F$?

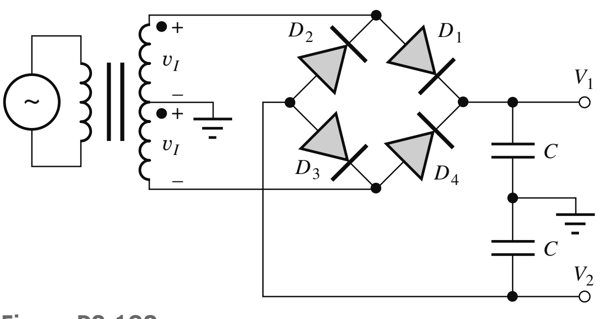


Figure P3.108

3.109. Simulate the rectifier circuit in Fig. P3.108 for $C = 100$ mF and $v_I = 40 \sin 2\pi(60)t$ with a 500-Ω load connected between each output and ground.

3.110. Repeat Prob. 3.97 if the full-wave bridge circuit is used instead of the rectifier in Fig. P3.97. Draw the circuit!

3.16 Rectifier Comparison and Design Tradeoffs

3.111. A 3.3-V, 15-A dc power supply is to be designed to have a ripple voltage of no more than 10 mV. Compare the pros and cons of implementating this power supply with half-wave, full-wave, and full-wave bridge rectifiers.

3.112. A 200-V, 0.5-A dc power supply is to be designed with less than a 2 percent ripple voltage. Compare the pros and cons of implementing this power supply with half-wave, full-wave, and full-wave bridge rectifiers.

3.113. A 3000-V, 1-A dc power supply is to be designed with less than a 4 percent ripple voltage. Compare the pros and cons of implementing this power supply with half-wave, full-wave, and full-wave bridge rectifiers.

3.17 Dynamic Switching Behavior of the Diode

*3.114. (a) Calculate the current at $t = 0^+$ in the circuit in Fig. P3.114. (b) Calculate I_F, I_R, and the storage time expected when the diode is switched off if $\tau_T = 8$ ns.

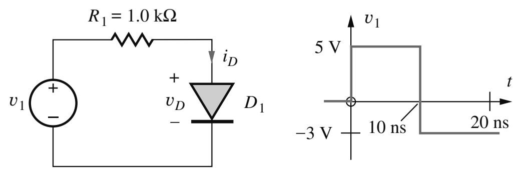


Figure P3.114

3.115. (a) Simulate the switching behavior of the circuit in Fig. P3.114. (b) Compare the simulation results to the hand calculations in Prob. 3.114.

*3.116. (a) Calculate the current at $t = 0^+$ in the circuit in Fig. P3.114 if R_1 is changed to 5 Ω. (b) Calculate I_F, I_R, and the storage time expected when the diode is switched off at $t = 10\ \mu s$ if $\tau_T = 250$ ns.

**3.117. The simulation results presented in Fig. 3.68 were performed with the diode transit time $\tau_T = 5$ ns. (a) Repeat the simulation of the diode circuit in Fig. 3.117(a) with the diode transit time changed to $\tau_T = 50$ ns. Does the storage time that you observe change in proportion to the value of τ_T in your simulation? Discuss. (b) Repeat the simulation with the input voltage changed to the one in Fig. P3.117(b), in which it is assumed that v_1 has been at 1.5 V for a long time, and compare the results to those obtained in (a). What is the reason for the difference between the results in (a) and (b)?

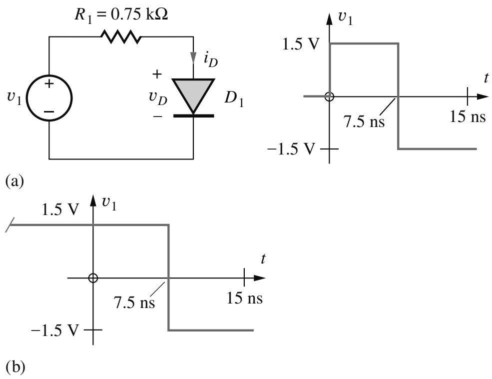


Figure 3.117

3.18 Photo Diodes, Solar Cells, and LEDs

*3.118. The output of a diode used as a solar cell is given by

$$I_C = 1 - 10^{-15}[\exp(40V_C) - 1] \text{ amperes}$$

What operating point corresponds to P_{max}? What is P_{max}? What are the values of I_{SC} and V_{OC}?

*3.119. Three diodes are connected in series to increase the output voltage of a solar cell. The individual outputs of the three diodes are given by

$$I_{C1} = 1.05 - 10^{-15}[\exp(40V_{C1}) - 1] \text{ A}$$
$$I_{C2} = 1.00 - 10^{-15}[\exp(40V_{C2}) - 1] \text{ A}$$
$$I_{C3} = 0.95 - 10^{-15}[\exp(40V_{C3}) - 1] \text{ A}$$

(a) What are the values of I_{SC} and V_{OC} for the series connected cell? (b) What is the value of P_{max}?

3.120. Write an expression for the total photo current i_{PH} for a diode having dc plus signal current components.

**3.121. The bandgaps of silicon and gallium arsenide are 1.12 eV and 1.42 eV, respectively. What are the wavelengths of light that you would expect to be emitted from these devices based on direct recombination of holes and electrons? To what "colors" of light do these wavelengths correspond?

**3.122. Repeat Prob. 3.121 for Ge, GaN, InP, InAs, BN, SiC, and CdSe.

术语对照

Anode	阳极
Avalanche breakdown	雪崩击穿
Bias current and voltage	偏置电流和电压
Breakdown region	故障区域
Breakdown voltage	击穿电压
Built-in potential(or voltage)	内置电位（或电压）
Cathode	阴极
Center-tapped transformer	中心抽头变压器
Conduction angle	导通角
Conduction interval	传导时间间隔
Constant voltage drop(CVD) mode	恒压降（CVD）模型
Cut-in voltage	接通电压
Depletion layer	耗尽层
Depletion-layer capacitance	耗尽层电容
Depletion-layer width	耗尽层宽度
Depletion region	耗尽区
Diffusion capacitance	扩散电容
Diode equation	二极管方程
Diode SPICE parameters(IS, RS,N,TT,CJO, VJ,M)	二极管SPICE参数
Filter capacitor	滤波电容器
Forward bias	正向偏压
Full-wave bridge rectifier circuit	电桥式整流电路
Full-wave rectifier circuit	全波整流电路
Half-wave rectifier circuit	半波整流电路
Ideal diode	理想二极管
Ideal diode model	理想二极管模型
impact-ionization process	碰撞电离过程
Junction capacitance	结电容
Junction potential	结面电位
Light-emitting diode(LED)	发光二极管（LED）
Line regulation	电源调整率
Load line	载重线

Load-line analysis	载重线分析
Load regulation	负载调整率
Mathematical model	数学模型
Metallurgical junction	冶金结
Nonideality factor(n)	非理想性因素（n）
Peak detector	峰值检波器
Peak inverse voltage(PIV)	峰值反向电压（PIV）
Photodetector circuit	光电探测器电路
Piecewise linear model	分段线性模型
pn junction diode	*pn*结二极管
proportional to absolute temperature(PTAT)	与绝对温度（PTAT）成正比
Q-point	Q点
Quiescent operating point	静态操作点
Rectifier circuits	整流电路
Reverse bias	反向偏压
Reverse breakdown	反向击穿
Reverse saturation current(I_S)	反向饱和电流（I_S）
Ripple current	波纹电流
Ripple voltage	波纹电压
Saturation current	饱和电流
Schottky barrier diode	肖特基势垒二极管
Solar cell	太阳能电池
Space charge region(SCR)	空间电荷区域（SCR）
storage time	存储时间
Surge current	冲击电流
Thermal voltage(V_T)	热电压（V_T）
transit time	通过时间
Turn-on voltage	开启电压
Voltage regulator	稳压器
Voltage transfer characteristic(VTC)	电压传递特性（VTC）
Zener breakdown	齐纳击穿
Zener diode	齐纳二极管
Zero bias	零偏差
Zero-bias junction capacitance	零偏压结电容